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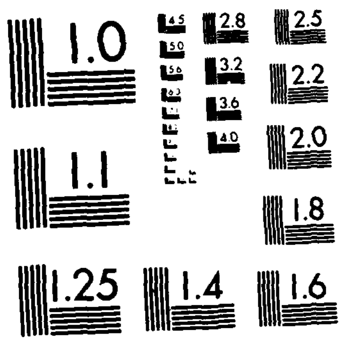

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AN ANALYSIS OF UNCERTAINTIES IN THE GROUND SHOCK ESTIMATION PROCESS

J. H. Wiggins Company, Inc.
1650 South Pacific Coast Highway
Redondo Beach, California 90277

22 June 1979

Final Report for Period 31 August 1978—30 May 1979

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A quantitative examination of some effects of uncertainty on the free-field ground shock estimation process is presented. Emphasis is placed on examining the modeling process and the associated uncertainties in the system response estimates with simplified models. Implications are presented regarding the optimal allocation of resources for obtaining samples and making property value measurements in order to reduce uncertainties in the overall free field ground shock estimation process. Associated sample size requirements are shown.

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CONVERSION FACTORS FOR U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASURE

To Convert From	To	Multiply By
Bar	Kilopascal	$1.000 \times 10^{+2}$
Foot	Meter	3.048×10^{-1}
Inch	Meter	2.540×10^{-2}
Kiloton	Terajoules	4.183
Megaton	Terajoules	4.183
Mile	Meter	$1.609 \times 10^{+3}$
Mile (Nautical)	Meter	$1.852 \times 10^{+3}$
Pound-Force/Inch^2	Kilopascal	6.894

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SECTION 1

INTRODUCTION

This report covers a portion of the efforts in an overall study that examined the role of uncertainty in the free field ground shock estimation process. The study was initiated by the Defense Nuclear Agency in order to address the following issues:

- o There is an increasing inquisitiveness about the presence of uncertainty throughout the ground shock estimation process and there is not a good understanding of the consequences of this uncertainty;
- o In the proposed implementation of new land-based systems, there is a large variety in soil conditions and geology. How can this variety be considered and controlled in the design process?
- o In view of the uncertainty in the output of ground shock measurements and simulations and, in view of the variability in the determination of material properties, what accuracy is warranted in the individual steps of the ground shock estimation process? Are some elements of the process over-worked while other elements need more emphasis, or is the uncertainty in some areas such that the computational error is small compared to the randomness of the problems?

These issues are of concern throughout the ground shock community which includes, among others, the soil property analyst, the designer of the ground facility, the simulation modeler, and the oversight agents for research and system expenditure. The community is somewhat fragmented in addressing the uncertainty issue and in correlating efforts. Therefore the objective of this study is to illuminate these issues and to analyze them in light of the offense and defense goals of the treatment of nuclear weapons by the Defense Nuclear Agency.

The interim report [Reference 1] discussed an approach to defining the uncertainties in the free field ground shock estimation process. This approach emphasizes an examination of the free field ground shock <u>modeling</u> process rather than an examination of the nuclear (and/or chemical) explosion test data on the basis of the limited size of these data bases.

The system is described by the sequence of steps shown in Figure 1-1. The modeler begins with the <u>real system</u> which, in this case, is composed of the weapon, the environment, and the propagation of energy. From this real system, <u>mathematical models</u> (representations) are created of elements of the system. For example, constitutive equations are developed for soil behavior under dynamic loading. The second box in the diagram depicts these mathematical representations of the behavior of the real system, or elements of the system.

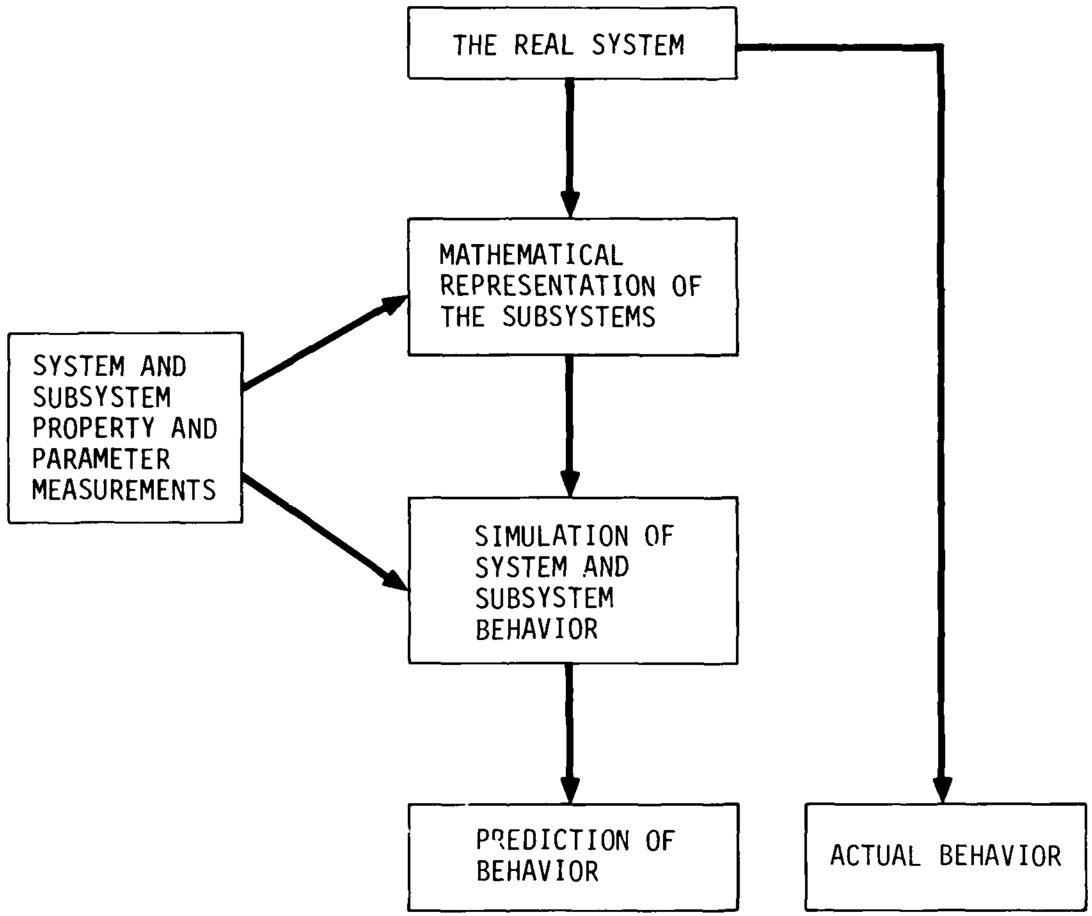


Figure 1-1. Representation of the real system.

The synthesis of the model into a computation involves simulation of the behavior of the real system. This simulation exercises the mathematical representations and should be structured to adequately reproduce or predict behavior within the narrow frame of a particular test configuration. Thus, the combined mathematical representations model the system, and simulation models a particular system response.

Both the mathematical representations and the simulation requires input in order to represent the real system. The data may be responses of system elements or physical measurements of parameters for use in the mathematical representations.

The modeling process, as shown in Figure 1-1, is very convenient in separating the various sources of error, uncertainty, and bias. Figure 1-2 shows how these uncertainties come from different sources and are due primarily to either the innate heterogeneity of the real system or due to breakdowns in the ability to maintain perfection in moving through the steps of the modeling process shown in Figure 1-1.

Reference 1 examined the free field ground shock estimation process from a systems analysis viewpoint and attempted to identify, in a qualitative manner, the various sources of uncertainty within the process. Many sources of uncertainty were identified but two were singled out as having the potential for being one of the major sources of uncertainty within the overall process and also being amenable at this time to quantitative analysis.

These two sources of uncertainty were: 1) The effects of innate heterogeneity of the physical and mechanical properties of earth materials on the measurement process, and 2) The set of assumptions that prevail throughout the various steps of the process that were characterized as the "average properties lead to average results" hypothesis.

There is little doubt as to the innate variability of certain physical and mechanical properties of earth materials. Table 1-1, which is summarized from Reference 2, indicates the typical spread in the value of the coefficient of variation of certain properties of sands, silts, and clays. A small value of the coefficient of variation, which is defined to be the ratio of the arithmetic standard deviation to the arithmetic mean of the property value, indicates a low degree of innate variability in the property value. Contrawise, a large coefficient of variation indicates a high degree of innate variability in the property value.

In addition to the overall innate variability of the properties of earth materials, Reference 3 indicates that spatial correlation of these property values may exist over the range of the site being investigated. The data presented in Reference 3 indicates spatial correlation distances for parameters such as listed in Table 1-1 that are of the order of a few hundred feet in horizontal extent and a few tens of feet in vertical extent. The potential existence of spatial correlation of material properties will have a definite impact on the sampling and testing schemes used to obtain the system measurements.

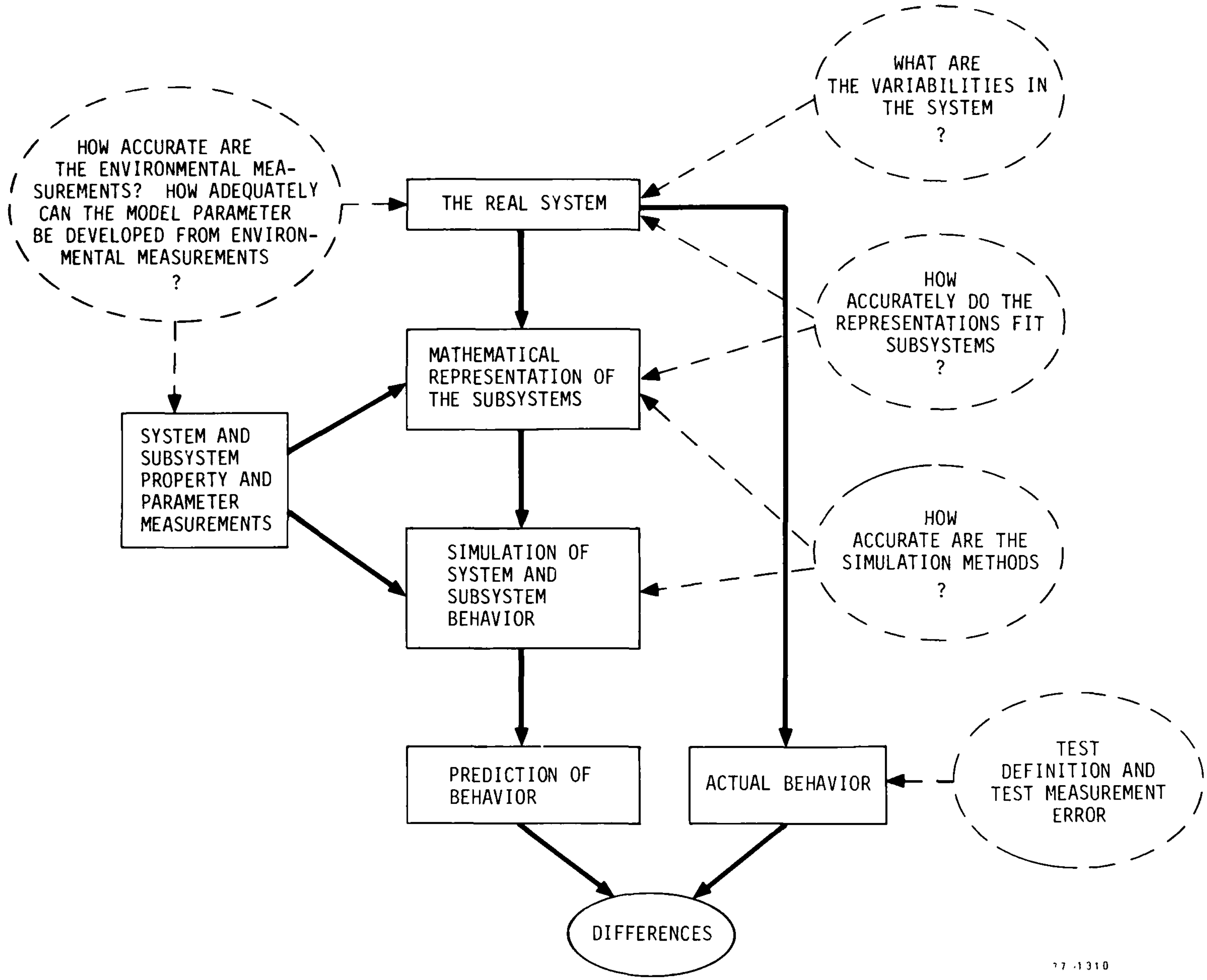


Figure 1-2. Uncertainty consideration in the prediction process

While the data shown in Table 1-1 indicate potentially large coefficients of variation for the physical and mechanical properties of earth materials, two problem areas exist. The first is that little if anything is known about the coefficients of variation of the physical and mechanical properties that exert the most influence on the free field ground shock estimation process. The second problem area is that the innate variability of the properties are of little interest in and of themselves. The real question of interest is how the innate variability of the properties interact with the model of reality being used to estimate the free field environment to produce uncertainty in this predicted environment.

This then leads into the question of the validity (or adequacy) of the "average properties lead to average results" hypothesis that prevails throughout the free field ground shock estimation process. This hypothesis is characterized by the assumptions tha site can be characterized by a set of homogeneous layers with horizontal bedding planes a that the physical and mechanical properties of these homogeneous layers are assigned "re presentative" (or average) values by the soils analysis based on a combination of laboratory and insitu testing.

The validity of the average properties/average results hypothesis can be questioned from two standpoints. First, that the hypothesis implicitly assumes either a relative degree of insensitivity of the value of the response with changes in the property value or small coefficients of variation for the property values. Second, the hypothesis innately assumes that extreme values of response are of interest only from the standpoint that they can occur rather than the frequency of occurrence of these extreme values. Both of these questions can only be answered from the standpoint of the decision maker as to whether use of the hypothesis leads to predictions of the free field environment that are adequate for system design and/or system survivability evaluation purposes.

The remainder of the report is divided into three sections. Section 2 provides the summary observations for the effort. Section 3 describes the analytical efforts and results of the examination of the adequacy of the "average properties lead to average results" hypothesis. Section 4 examines the question of sampling and testing strategies for obtaining system measurements in the face of the innate variability of earth materials and the absence or presence of spatial correlation of the physical properties of earth materials.

Table 1-1. Coefficient of variation ranges for selected soil parameters.

Parameter Type	Selected Example	Coefficient Of Variation Range(%)
Volumetric/Gravimetric	Specific Gravity	1 → 25
	Void Ratio	13 → 30
Compressibility	Recompression Ratio	25 → 80
	Compression Index	25 → 55
Strength	Friction Angle	5 → 15
	Unconfined Compression Strength	30 → 85

*Summarized from Chapter 10 of Reference 2.

SECTION 2

SUMMARY OBSERVATIONS

The "average properties lead to average results" hypothesis was found to lead to a biased estimate of the mean free field response for the one-dimensional vertical airslap in the superseismic airblast region problem examined in this effort. For both the case of peak vertical velocity and peak vertical displacement, the hypothesis leads to mean response values that are consistently lower than those derived without the use of the hypothesis. The degree of bias was found to decrease with increasing depth for both response parameters ranging from a maximum of about 15 percent near surface to a minimum of about 1 percent at depths near 150 feet which was near the maximum depth monitored in the analysis.

During the Monte Carlo simulation phase of the effort, it was found that the peak response values were related to the value of a single material parameter value according to a mathematical relationship of the form: peak response proportional to (property value)$^{\alpha}$, where the value of the exponent α and the constant of proportionality depended on the response being considered and the depth below the ground surface. The investigation of the adequacy of the hypothesis was generalized to include all relationships of this form. The average properties/average results hypothesis was found to be a biased estimator of the mean response for all relationships of this type. The degree of bias was found to depend upon the value of the exponent (α) and the coefficient of variation of the property value.

The hypothesis was found to lead to a modest overestimate of the mean response for all values of the exponent between 0 and 1.0 and all coefficients of variation of the property value in the range of 0 to 1.0. Thus, for example, the hypothesis will lead to a modest overestimate of the wave propagation velocity which varies as the positive square root of the constrained modulus. For all other values of the exponent, the hypothesis was found to lead to a consistent underestimate of the mean response. As previously mentioned, the degree of bias depends on the value of the exponent and the coefficient of variation of the property value. For example, with a coefficient of variation of 0.7 (which is approximately the maximum likelihood estimate for the loading modulus of the dry sand considered in the Monte Carlo simulations) and exponents in the ranges of 1.0 to 1.5 and 0 to -0.5 (Note the symmetry around +0.5), the hypothesis will produce an estimate of the mean response that is a maximum of about 15% low. On the other hand, for the same ranges of exponents and a coefficient of variation of the property of unity (an admissible value from the dry sand data), the hypothesis will lead to estimates of the mean response that are up to a factor of 2 too low.

Overall, this test of the validity of the "average properties lead to average results" hypothesis when applied to the free field ground shock estimation process led to

mixed results. In the case of the one-dimensional vertical airslap problem, the hypothesis led to predictions of mean response that can be argued as "certainly being within the accuracy of the input data." On the other hand, generalization of the form of the relationships found in the vertical airslap problem showed that the hypothesis always leads to a biased estimate with a degree of bias that is determined by the particular relationship between response and property value and the coefficient of variation of the property value.

Extrapolation of these results to problems where the response values depend on more than one property value suggests that the validity of the average properties/average results hypothesis should be investigated on a case by case basis. In general, it would be expected that if the response was relatively insensitive to parameter value or the parameter value was known to have a small coefficient of variation, the hypothesis would produce mean response values that were within an acceptable degree of bias. If these conditions are not met, then the hypothesis is probably inadequate and the mean response will have to be estimated using other analytical techniques or, as a last resort, Monte Carlo simulations.

Discussion to this point has assumed that the material property values have no uncertainties. This, in general, will certainly not be the case. Sample size limitations and uncertainties introduced by the sampling and testing process will produce uncertainties in both the estimate of the mean of the property value and the estimates of the variance of the property value. For example, the available data for the uniaxial stress/strain characteristics of the dry sand material used in the hypothesis test consisted of 15 stress/strain relationships. These data lead to uncertainties (at the 0.9 confidence level) of about a factor of approximately 1.5 in the estimate of the mean loading modulus and about a factor of 2 in the estimate of the variance of the loading modulus.

With the caveat of "beware of systematic (or bias) uncertainties introduced within the sampling and testing process", the uncertainty in the estimate of the mean property value can be reduced to any desired level of precision simply by increasing the number of samples that are tested. The existence of random sampling and testing uncertainty components merely modifies the sample size requirements. Similarly, the uncertainty in the estimate of the variance can be reduced by increasing the sample size, but the estimate of the variance will include components from both the innate variability of the property value and the random sampling and testing errors.

The degree of uncertainty in the parameters of the property value distribution are, however, of little interest in themselves. The parameters of real interest are the uncertainties in the free field ground shock response parameters. The estimation of which requires consideration of both the uncertainties in the property value distribution parameters and how these uncertainties interact with the model of reality being used to estimate the responses.

For the vertical airslap problem examined in this effort, the variance-reducing characteristics of the model of reality used in the analysis leads to uncertainties in the mean modulus value in the neighborhood of 20-30% producing uncertainties in the near-surface peak velocities and peak displacements that are of the same order as the innate bias of the average property/average results hypothesis. Sample sizes of the order of a small multiple of ten are expected to be adequate to produce uncertainties in the mean property value that are of this level if there are no spatial correlations of material property values. The existence of spatial correlation should roughly double the number of samples required to produce this level of uncertainty in the mean property value.

The estimates of the mean and variance of the loading modulus of the dry sand material also produced extreme values of the near surface responses that differed from the mean values of the responses by nearly a factor of 2 at the 0.9 conditional confidence level. (Note that these confidence bounds are conditional on the estimate of the means, the variance and the assumed form of the distribution function for the property values being correct.) These limits are in themselves quite uncertain since the estimate of the variance of the property value distribution is uncertain by nearly a factor of 2 at the 0.9 confidence level.

Reducing the uncertainty in the estimate of the variance of a property value is a much more formidable task than reducing the uncertainty of the estimate of the mean of the property value. Sample sizes of the order of 100 are required when no spatial correlation of property values exist to produce uncertainties in the extreme values of response that are of the same order as the inherent bias of the average property/average results hypothesis when applied to the vertical airslap problem. The existence of spatial correlation of the property values may increase the number of samples required by the order of a factor of 5. Again, this estimate of the variance will include random sampling and testing error components.

Extension of these results to other portions of the free field ground shock estimation process should be done with extreme care. If the uncertainties in the estimates of the parameter values interact with the model of reality in the variance-reducing manner, then the previously-mentioned ranges of sample size requirements are applicable. On the other hand, if the uncertainties interact in a variance-magnifying manner, the sample sizes required to maintain a fixed level of uncertainty in the response values will have to be significantly increased.

The existence of spatial correlation of material property values forces some special consideration when planning the exploration of an area such as, perhaps, a "MX Valley". An optimal allocation of resources between obtaining samples from different areas and making property value measurements occurs that minimizes the uncertainty in the estimate of the mean property value that is independent of total exploration costs.

The fraction of an exploration budget allocated to property value measurements depends only on two parameters: the ratio of the cost of obtaining samples within a subarea (in the limit boreholes) to the cost of making a property value measurement, and the ratio of the components of the total variance which are referred to as the local variance and the variance of the means. When the cost ratio is low, as is probably the case of the CIST tests, the optimal allocation of resources involves making one measurement in each subarea. As the value of the cost ratio becomes larger, the optimal allocation of resources involves making an increasingly large number of measurements on samples taken from an increasingly smaller number of subareas. Similarly, for a fixed cost ratio the optimal allocation of resources involves a small number of samples from a large number of subareas when the local variance (which includes random sampling and testing error components) is small when compared to the variance of the means. As the ratio of the variance increases, the optimal allocation involves increasing the number of measurements per subarea at the expense of decreasing the number of subareas investigated.

Generalizing these results to the case where more than one property value is of interest suggests a conflict may arise between the optimal allocation of resources for the estimation of a property value such as near surface loading modulus and the estimation of parameters such as the depth profile for the valley under investigation. While no firm data exist that support the conclusion, the nature of the optimal allocation scheme suggests that compromise allocations can be arrived at that either maintain the precision of the estimates of the property values at modest increases in the total exploration costs or maintain the total exploration costs at modest decreases in the precision of the estimates of the property value.

SECTION 3

TESTING THE AVERAGE PROPERTIES LEAD TO AVERAGE RESULTS HYPOTHESIS

Throughout the free field ground shock estimation process, it is common to make a series of assumptions that can be characterized as the "average properties lead to average results" hypothesis. This hypothesis, which manifests itself in the assumption that the site under consideration can be represented by a series of homogeneous layers with horizontal bedding planes, implicitly assumes that any effects of the innate heterogeneity of the physical and mechanical properties of earth materials will average out. Thus, using average values of these properties in the prediction process are hypothesized to produce predictions of the average response.

Devising a method of testing the validity of a hypothesis such as "average properties lead to average results" is difficult from the formal logic standpoint since feasible tests must be based on simulations of reality. The results of such test must, therefore, be viewed as necessary but not sufficient, conditions for accepting or rejecting the hypothesis being tested.

The mechanism initially chosen for testing the validity of the average properties/average results hypothesis was to utilize a one-dimensional simulation of vertical motion in the superseismic airblast region and to compare the mean peak velocities produced by a Monte Carlo simulation of the problem with the peak velocities calculated using the "average properties lead to average results" hypothesis for a particular site representation. Based on intermediate results obtained in this Monte Carlo simulation analysis, this was generalized to cover the question of the adequacy of the hypothesis when peak responses vary with material parameters according to certain forms of mathematical relationships.

3-1 MONTE CARLO SIMULATIONS

Two criteria were used in arriving at the site geology to be simulated. The first criteria was that the geology must be relatively simple so that a simple wave propagation code could be used along with a Monte Carlo driver program. The second criteria was that there should exist a number of measurements of the physical and/or mechanical properties of the site materials sufficient to estimate both the average values of the properties and the innate variability of the properties.

After a considerable literature search, the site configuration shown in Figure 3-1 was chosen. This configuration represents the upper portion of one of the representative potential MAP sites given in Reference 4. Figure 3-2, which is taken from Reference 4, illustrates several of the uniaxial strain relationships of vertical stress versus vertical strain for dry sand samples from the upper layer of the site profile. Overall, a total of 15 of these relationships were available for the estimate of the statistical properties of dry sand.

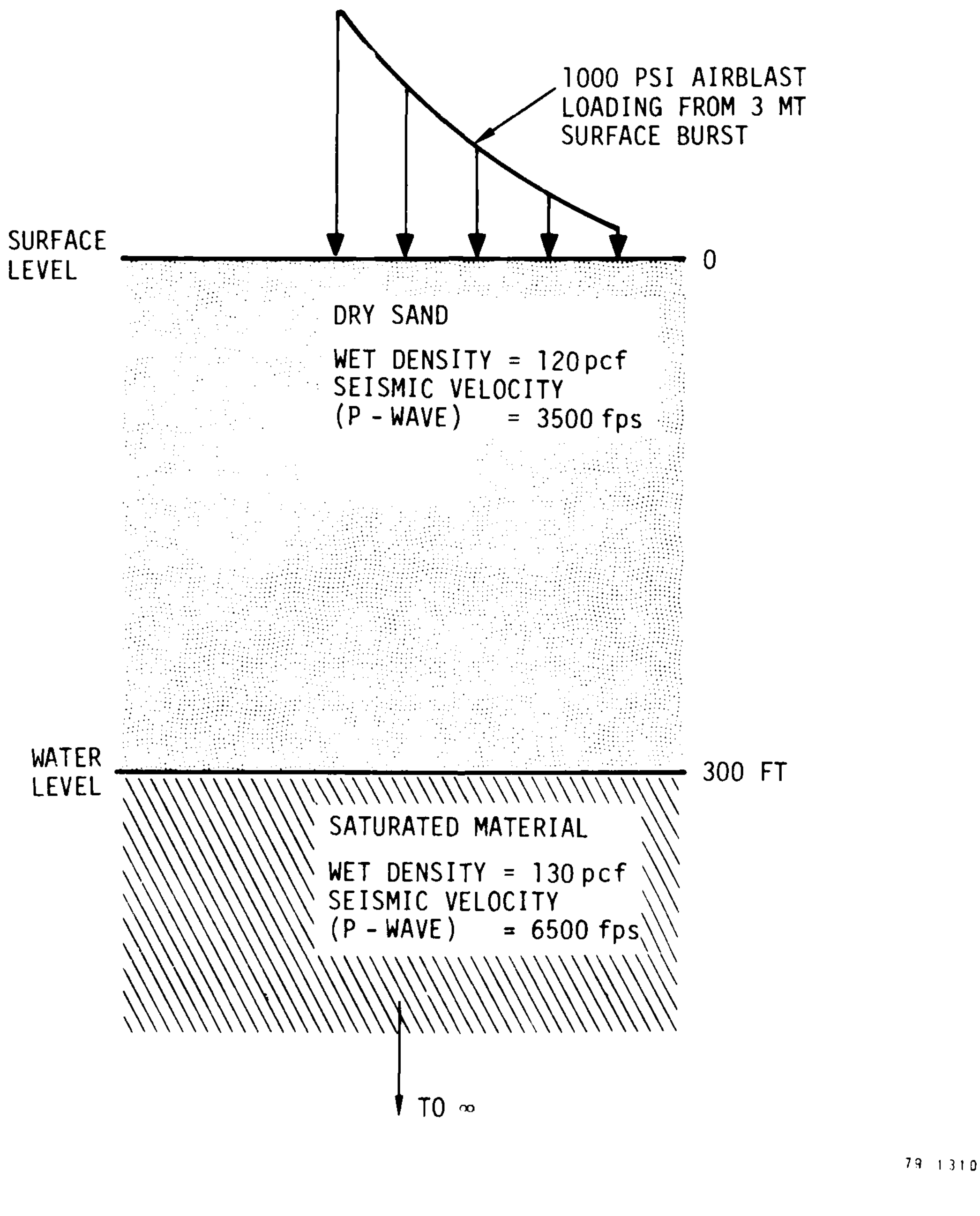


Figure 3-1. Site profile considered.

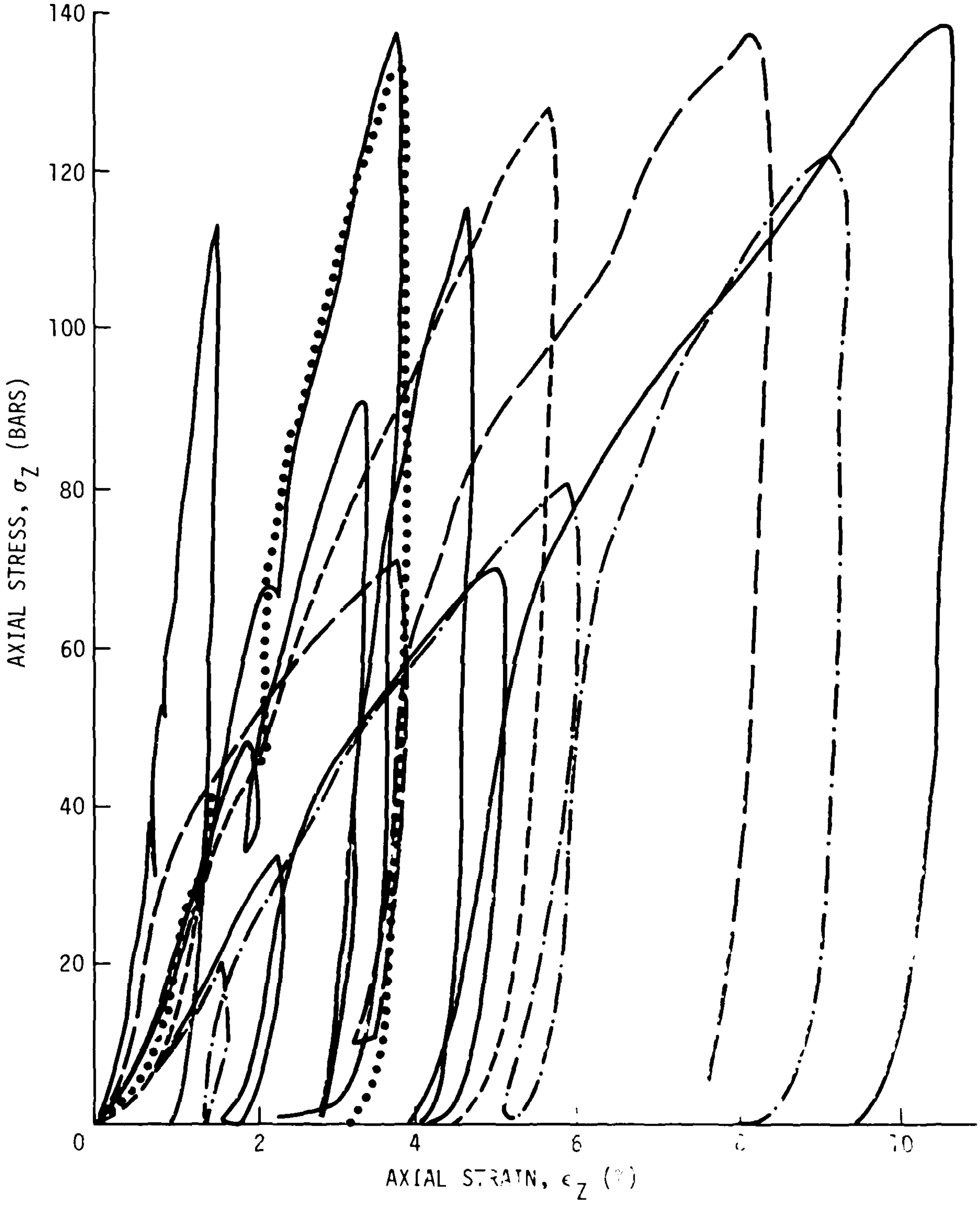


Figure 3-2. LBGR undisturbed sands - typical WES US test data.

Figure 3-3a shows the representative properties of the dry sand as derived by the Waterways Experiment Station (WES) and reported in Reference 4. The representative properties are characterized by a loading modulus of about 2.60 Kbars at higher strains and on unloading modulus of 85 Kbars. Figure 3-3b illustrates the statistical representation of the properties. This representation assumes that the initial loading and the unloading modulus are the same as those of the WES representative properties. The loading modulus at the higher strains were generated by fitting the modulus values at 1% strain derived from the 15 available stress-strain relationships to a log normal distribution. The maximum likelihood estimate of the median of the modulus distribution is 2.03 Kbars while that of the standard deviation is 0.64. This leads to a mean modulu value of 2.53 Kbars which is within a few percent of the WES representative property value.

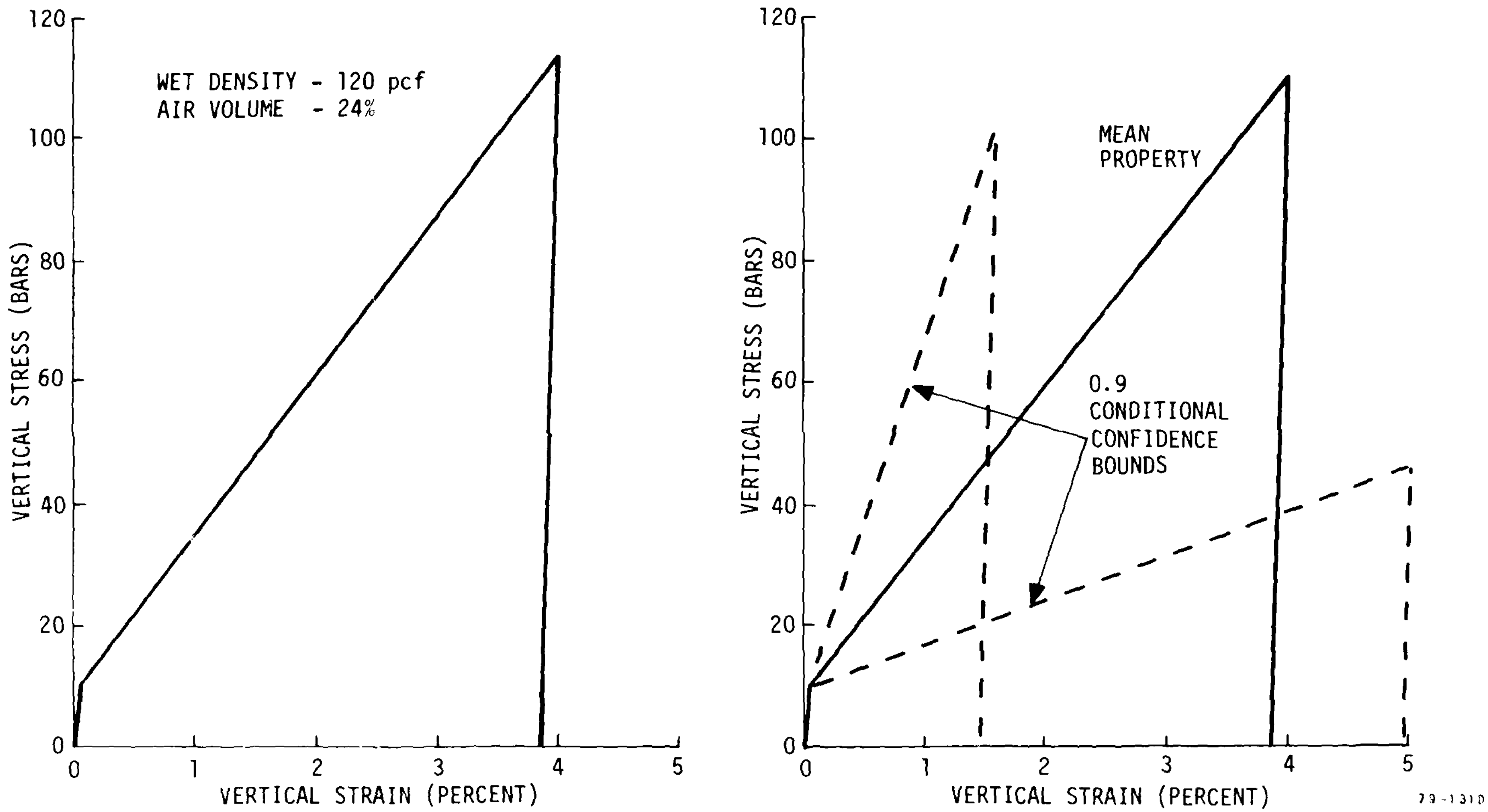
The wide spread in modulus values and the limited number of data points leads to quite large confidence regions for the estimates of the true mean modulus and the true coefficient of variation. The 0.9 confidence limits for the median modulus are 1.55 Kbars and 2.76 Kbars while the same confidence limits for the standard deviation are 0.49 and 0.93. This leads to 0.9 confidence bounds for the true coefficient of variation of roughly 0.5 and 1.2 compared to a nominal value of about 0.7.

Since the question at hand was testing the "average properties lead to average results" hypothesis rather than propagation of uncertainty through the model, the best estimate values of the mean modulus and the standard deviation were assumed to be a reality. This leads to the 0.9 conditional confidence bound stress-strain relationships shown in Figure 3-3b which has upper loading modulii that differ from the mean value by nearly a factor three.

The ONED code [Reference 5] was chosen as the wave propagation code for the Monte Carlo simulations. This choice was made solely on the basis of minimizing the computer time requirements by avoiding intermediate input/output operations during the Monte Carlo simulation cycles. The ONED code was modified to act as a subroutine to the driver program whose additional functions were to generate the modulus value to be used for each simulation cycle from a log-normal distribution, to calculate the overpressure vs time waveforms using the methodology of Reference 7, and to perform the calculation of the mean and the variance of the peak velocity values generated during the Monte Carlo simulation cycles.

The overpressure wave form used was that of the 1000 psi contour from a 3 MT surface burst. The rise time for the wave form was set to satisfy the numerical stability requirements posed by the smallest modulus value likely to be encountered during the set of Monte Carlo cycles. Maximum simulation time for each cycle was determined by the time required for the velocity at 150 foot depth to reach its maximum value.

Figure 3-4 shows the results of a 41-sample Monte Carlo simulation in terms of the cumulative distribution of peak velocity layers at 1.67 foot depth in the dry sand layer.



a) WES REPRESENTATIVE PROPERTIES FOR DRY SAND

b) STATISTICAL REPRESENTATION OF PROPERTIES

Figure 3-3. Models of uniaxial stress/strain characteristics of dry sand.

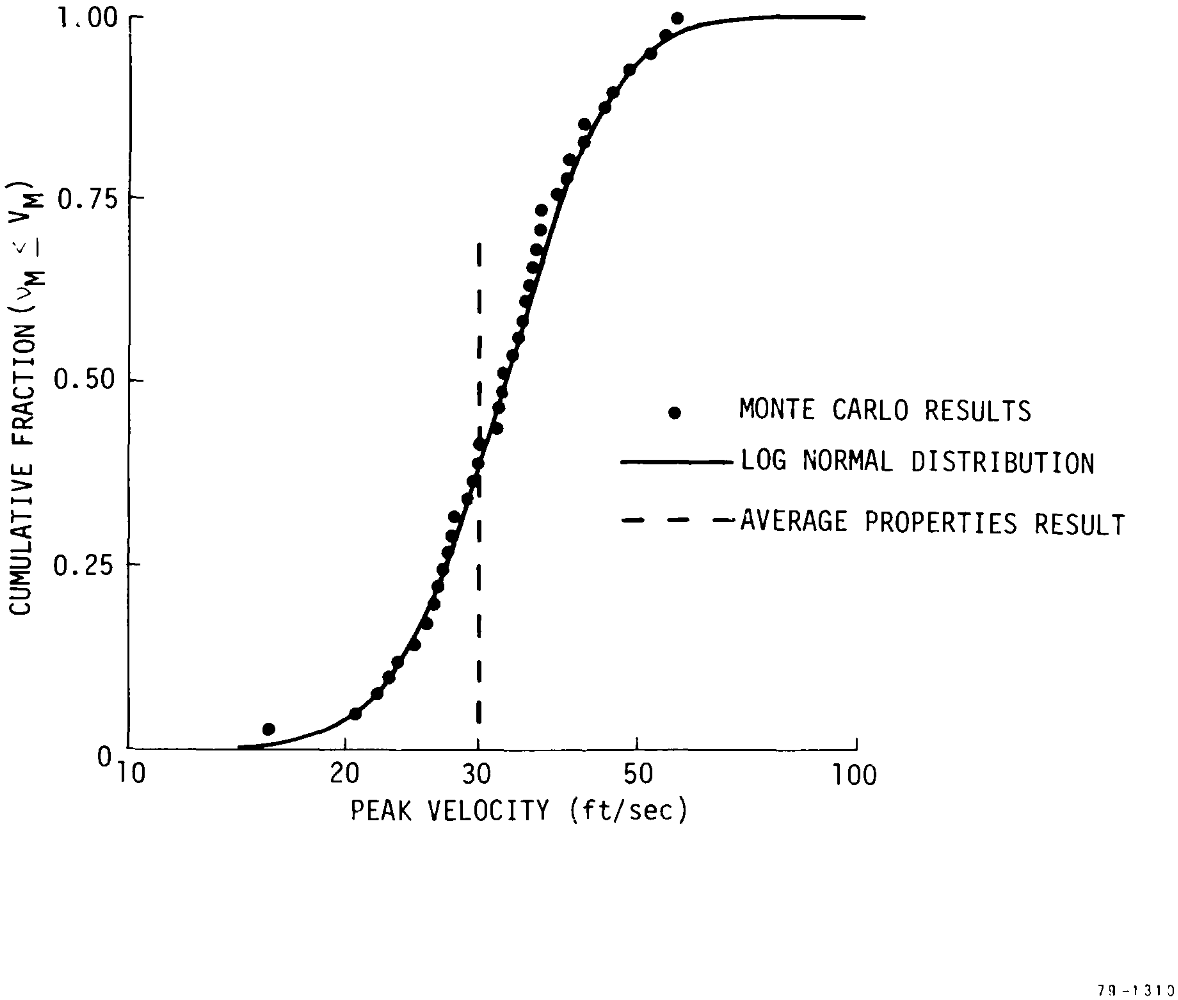


Figure 3-4. Cumulative distribution of peak velocity values for 1.67 ft. depth.

Also shown are the best fit to the data using a cumulative log normal distribution and the peak velocity calculated using the average properties stress-strain relationship. The Monte Carlo results for this depth have a median peak velocity of 32.5 ft/sec, a mean peak velocity of 33.7 ft/sec and a coefficient of variation of about 0.28. The peak velocity calculated using the average properties is 30.0 ft/sec, i.e., about 10% lower than the mean peak velocity from the Monte Carlo results. A "goodness to fit" test was used to test the hypothesis that the Monte Carlo results were samples from a log normal distribution. For these data, the hypothesis of log normality is acceptable at greater than the 0.95 confidence level.

Figure 3-5 shows similar information for the depth of 21.67 feet. For this depth, the Monte Carlo results have a median peak velocity of 26.9 ft/sec, a mean peak velocity of 27.5 ft/sec and a coefficient of variation of about 0.21. The log normality of the peak velocity values was again strongly supported by the goodness of fit test. The peak velocity calculated using the average properties is 25.4 ft/sec which is about 8 percent lower than the mean peak velocity from the Monte Carlo simulations.

Overall, the Monte Carlo results at all depths in the dry sand between the near surface and 160 foot depth showed the mean and median peak velocities decreasing with increasing depth, strong support of the hypothesis of log normality of the peak velocity values, and coefficients of variation that decreased with increasing depth. In all cases, the peak velocities calculated using the average properties were lower than the means of the Monte Carlo results. The percentage difference between the two values, however, decreased with increasing depth.

The log normality of the peak velocity values suggests that there is, perhaps, a simple, functional relationship between peak velocity and the upper loading modulus which was assumed to have a log normal distribution. Figure 3-6 shows the relationship between the peak velocity values at 1.67 foot depth calculated in the Monte Carlo simulations and the upper loading modulus value used in the particular simulation cycle. Also shown is the least squares regression line relating the peak velocity to the upper loading modulus. This relationship is of the form.

$$V_{max} = aM^{\alpha} \tag{1}$$

where V_{max} is the peak velocity, M the upper loading modulus, and a and α are constants. For the case shown, α has a value of approximately -0.48 and the regression line accounts for almost all the variability in the peak velocity values. Similar relationships were observed at other depths. The value of the exponent α was found to decrease from roughly -1/2 to roughly -1/16 over the depth interval of surface to approximately 160 feet.

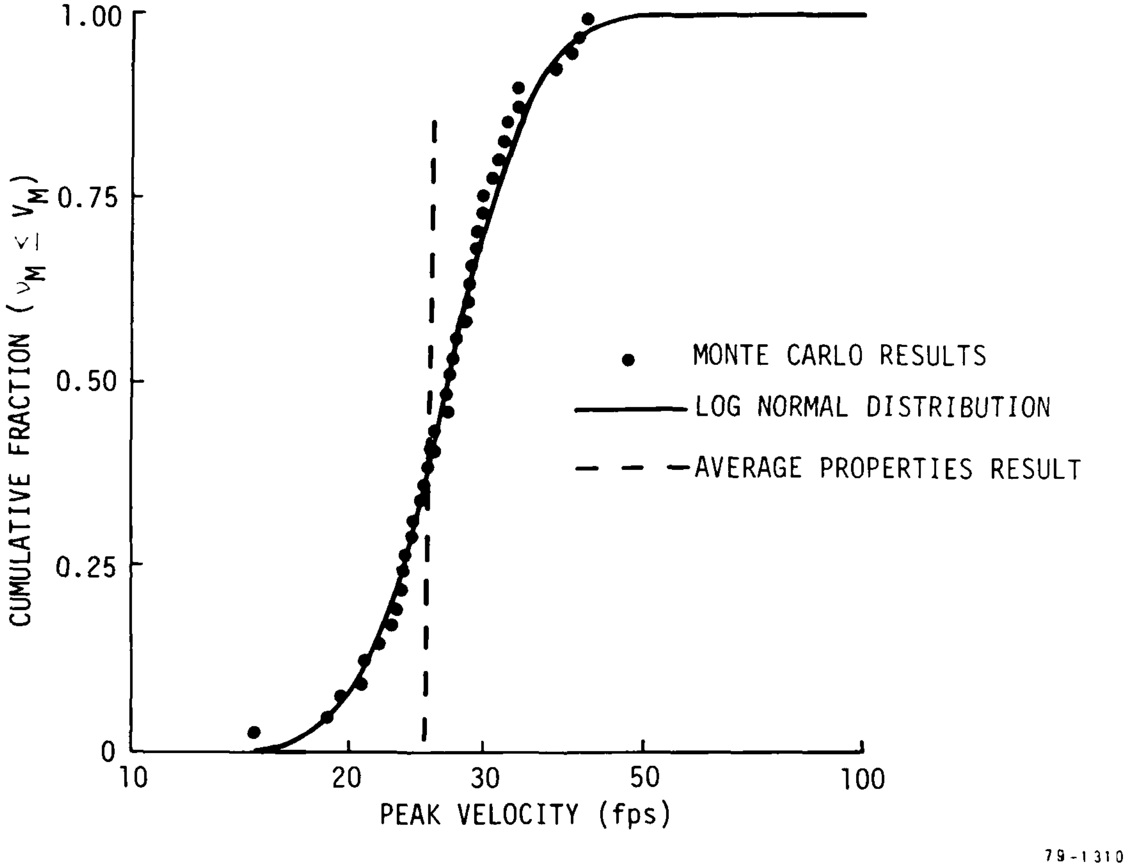


Figure 3-5. Cumulative distribution of peak velocity values for 21.67 ft depth.

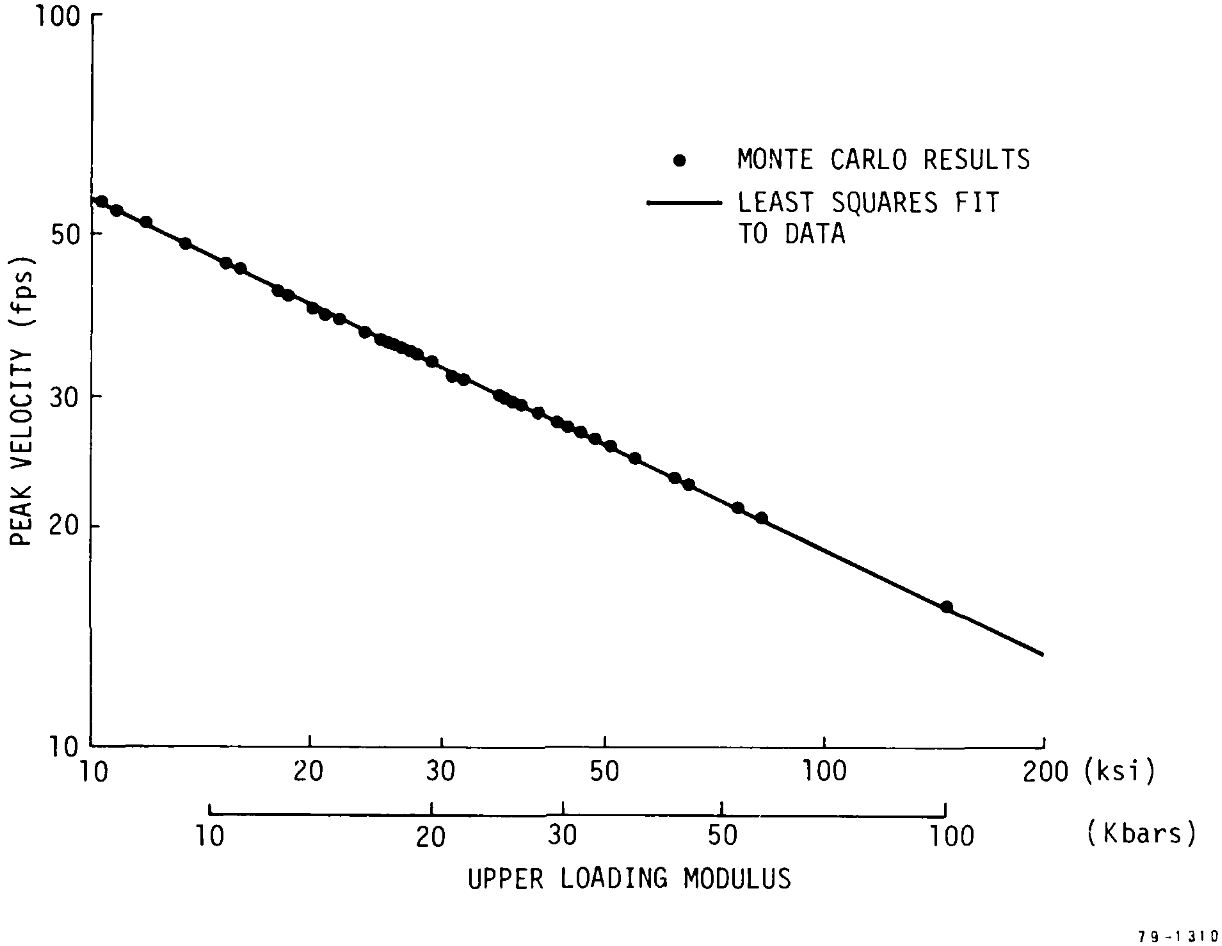


Figure 3-6. Empirical relationship between velocity and modulus for 1.67 ft. depth.

The existence of these relationships greatly simplified the test of the average properties/average results hypothesis since it negates, in certain cases, the necessity of large Monte Carlo sample sizes by permitting closed-form analytical solutions. Even in those cases where Monte Carlo techniques are required, they can be performed using the simple analytical relationship rather than the full-up simulation.

Before proceeding, it is useful to enumerate certain properties of log normal distributions. Since the upper loading modulus was assumed to be log normally distributed, we have

$$\text{Median Modulus} = M_{50} \tag{2}$$

$$\text{Standard Deviation} = \beta \tag{3}$$

and the relationships

$$\text{Mean Modulus} = M_{50} \exp\left[\frac{1}{2}\beta^2\right] \tag{4}$$

$$\text{Coefficient of Variation} = \sqrt{\exp \beta^2 - 1} \approx \beta \tag{5}$$

Because of the indicated relationship between the peak velocity and the upper modulus, the peak velocity values will also be log normally distributed with

$$\text{Median Velocity} = V_{50} = aM_{50}^{\alpha} \tag{6}$$

$$\text{Standard Deviation} = \gamma = |\alpha| \cdot \beta \tag{7}$$

$$\text{Median Velocity} = V_{50} \exp\left[1/2\gamma^2\right] \tag{8}$$

$$\text{Coefficient of Variation} = \sqrt{\exp\left[\alpha^2\beta^2\right] - 1} \approx |\alpha|\beta \tag{9}$$

and the 0.9 conditional confidence bounds on the peak velocity values will be given by

$$V_{bounds} = V_{50} \exp\left[\pm 1.645\gamma\right] \tag{10}$$

where the plus sign gives the upper bound and the minus sign gives the lower bound. For comparison, the average properties' peak velocity will be given by

$$V_{ap} = a \left[M_{50} \exp \left[1/2\beta^2 \right] \right]^{\alpha} \tag{11}$$

$$= V_{50} \exp \left[1/2\alpha\beta^2 \right]$$

which is smaller than the mean peak velocity for all negative and some positive values of α.

The relationships between the parameters of the upper loading modulus distribution are graphically illustrated in Figure 3-7 for the case of 1.67 foot depth. In both Figure 3-7a, which deals with the means and medians, and Figure 3-7b, which deals with the confidence limits, the relationship between peak velocity and upper loading modulus shown in Figure 3-6 is represented by the long dashed line.

Addressing first the cases shown in Figure 3-7a, the mean upper modulus value of 2.53 Kbars (36.7 Ksi) corresponds to an average properties' peak velocity value of 30.1 fps whereas the median modulus value of 2.07 Kbars (30.0 Ksi) leads to a median peak velocity of 33.1 pfs. The standard deviation of the log normal upper modulus distribution is about 0.63 while the exponent of the peak velocity upper modulus relationship is -0.48 for this depth. This leads to a standard deviation of the peak velocity distribution of about 0.30 and a mean peak velocity is 34.6 fps at this depth. (Note again that the mean peak velocity does not correspond to any single descriptor of the upper loading modulus distribution.)

In Figure 3-7b, the upper bound upper modulus value of 5.89 Kbars (85.4 Ksi) produces a lower bound for the peak velocity distribution of 20.1 fps. Similarly, the lower bound modulus value of 0.73 Kbars (10.6 Ksi) produces an upper bound peak velocity value of 54.5 fps. Thus, although the 0.9 confidence bounds for the upper modulus distribution differ by a factor of 8.1, the 0.9 confidence bounds for the peak velocity distribution differ by a factor of 2.7 at this depth.

Figure 3-8 shows the effect of depth on the calculated peak velocity values. The average properties peak velocity value is lower than the mean velocity value at all depths. The percentage difference between the two values, however, decreases with increasing depth. Near surface, the two peak velocity values differ by some 13 percent. Near 150 foot depth, the two values differ by some 1.5 percent. Notice also that the magnitude of the difference between the lower and upper confidence bounds on the peak velocity also decrease with increasing depth. Near surface they differ by a factor of 8.1 while near 150 foot depth the two bounds differ by a factor of 1.15. Both of these behaviors are due to the previously mentioned decrease with increasing depth of the exponent in the relationships between peak velocity and upper loading modulus.

Figure 3-9 shows the relationship between peak displacement at 1.67 foot depth and upper loading modulus that was derived from a series of non-Monte Carlo runs of the ONED

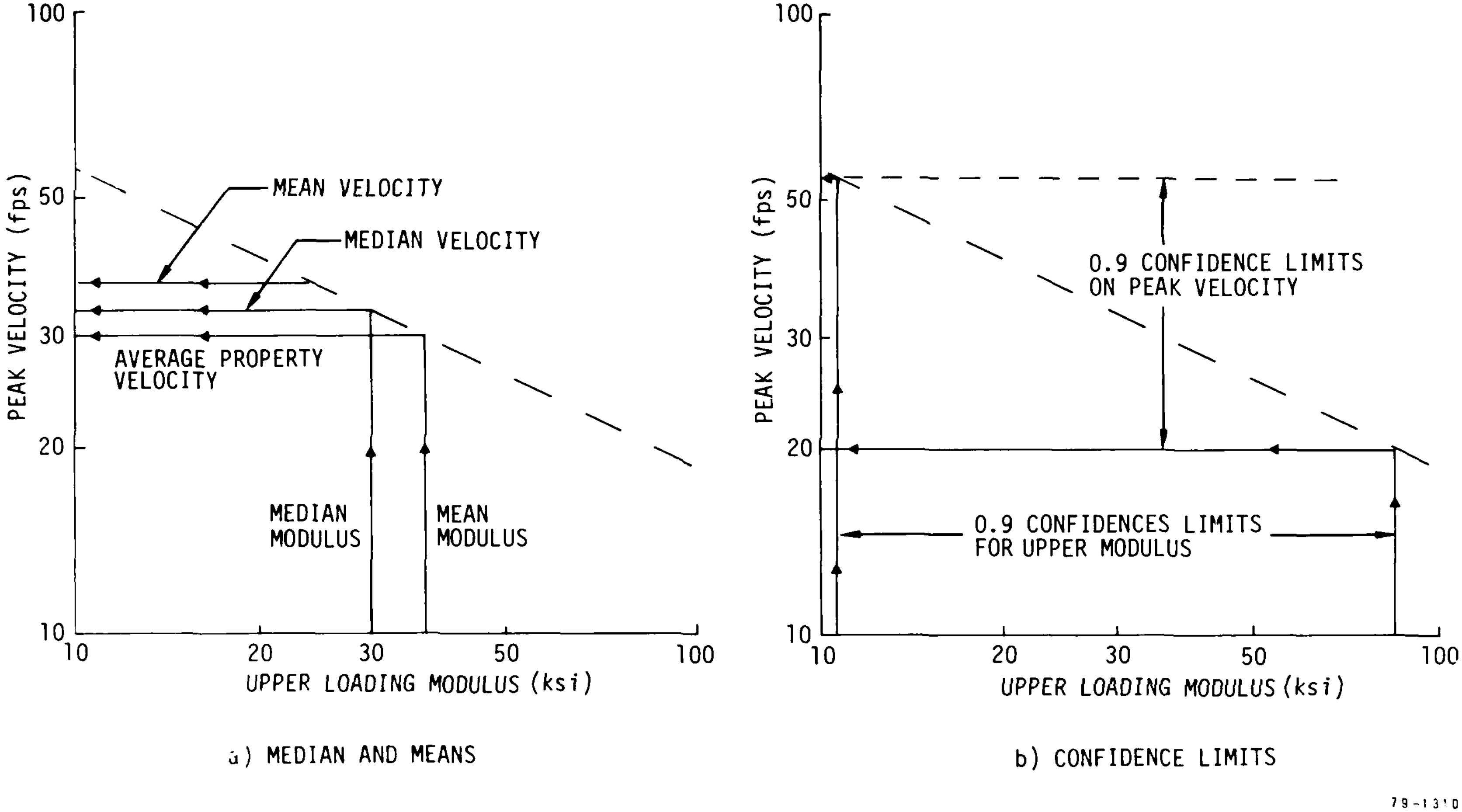


Figure 3-7. Graphical construction of velocity values for 1.67 ft. depth.

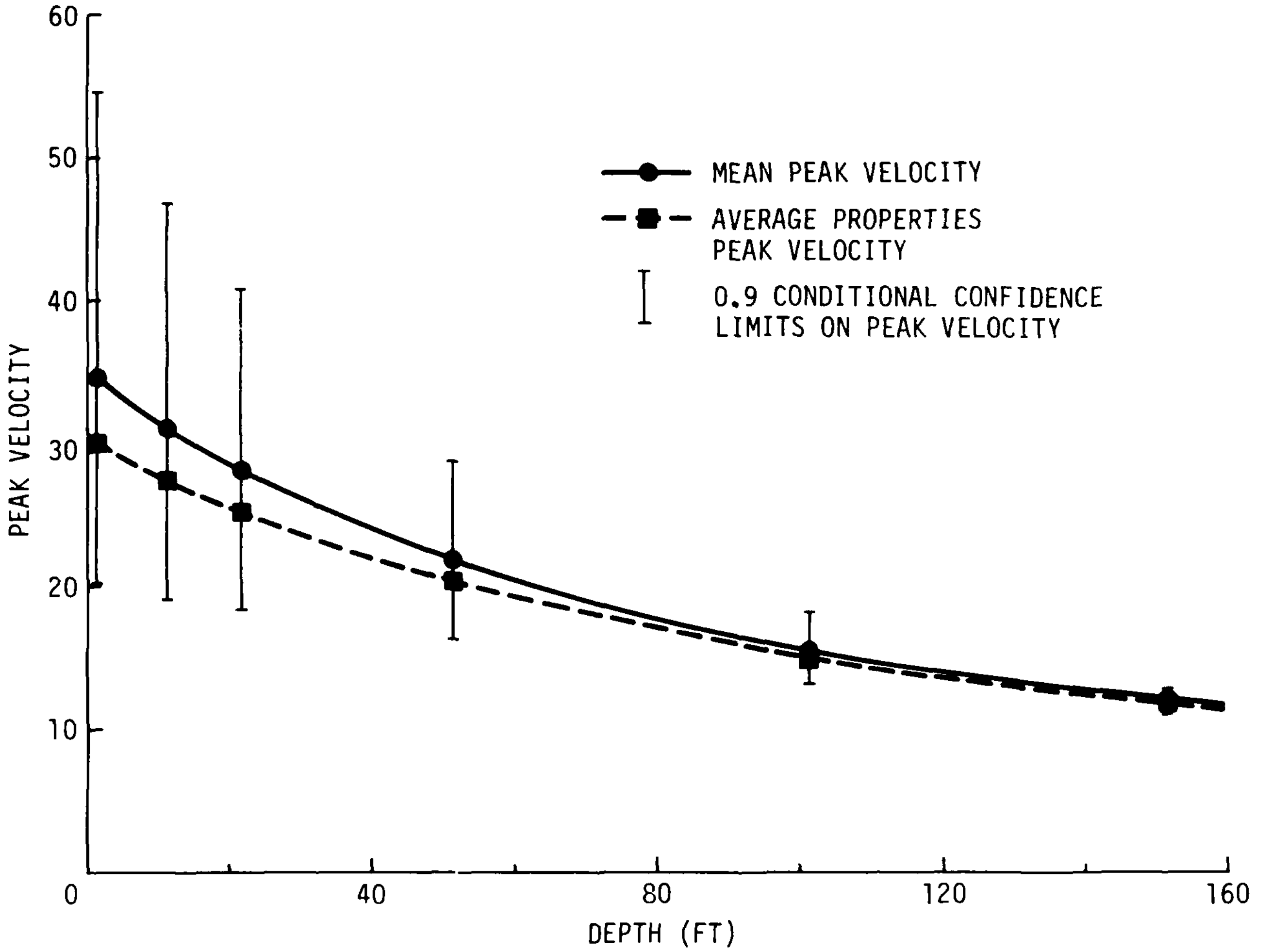


Figure 3-8. Effect of depth on calculated peak velocities.

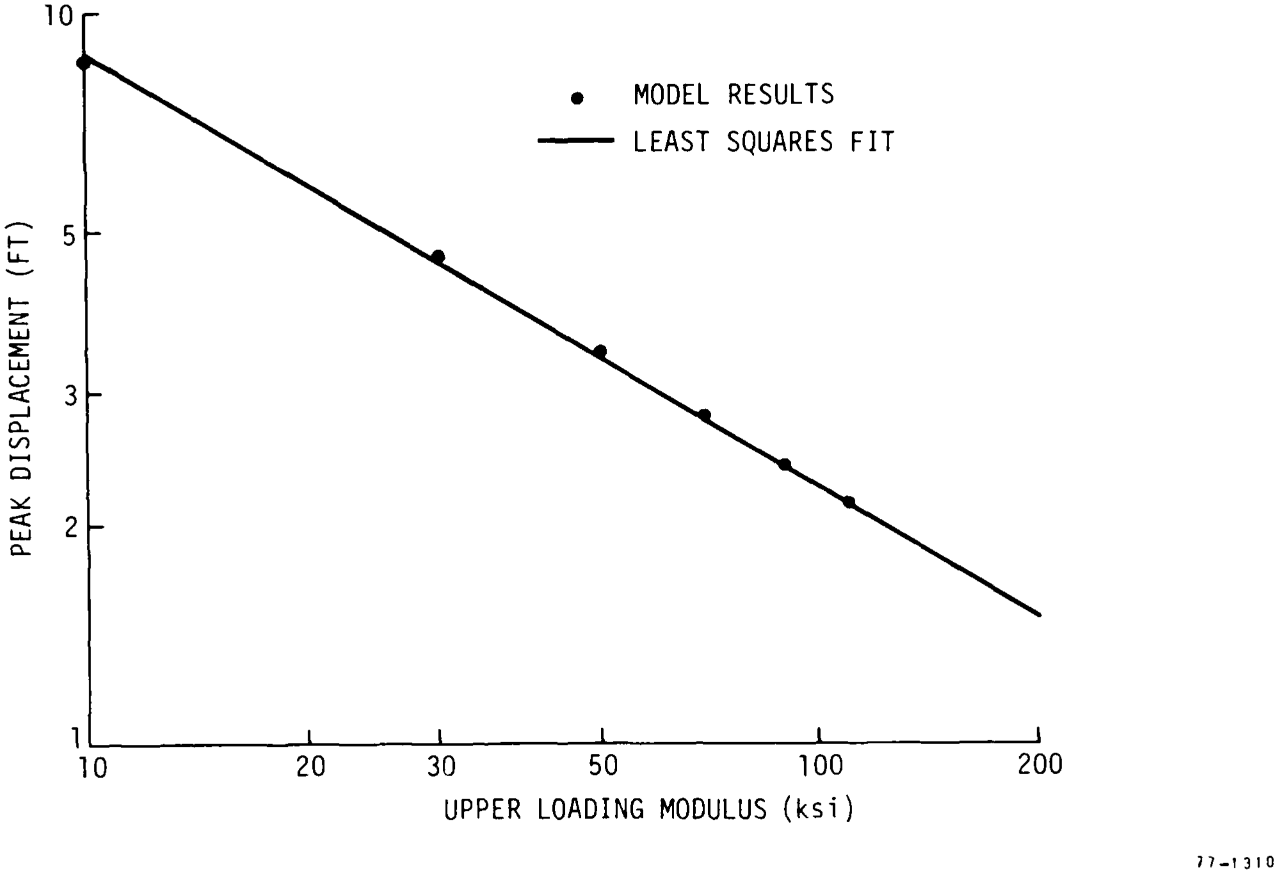


Figure 3-9. Empirical relationship between peak displacement and upper loading modulus for 1.67 ft. depth.

model. The least squares regression fit to these data points leads to a relationship of the form

$$D_{max} = aM^{-\alpha} \tag{12}$$

with an exponent of about 0.59 for this depth. The peak displacement data for other depths produced relationships between peak displacement and upper loading modulus that were similar in form. As was the case of the peak velocity relationships, the exponent decreases with increasing depth, but at a slower rate. The value of the exponent varies from about 0.59 for the 1.67 foot depth to 0.31 for depths near 150 feet.

Figure 3-10 shows the effect of depth on the various peak displacement values. As was the case with peak velocity, the average properties result is lower than the mean peak displacement at all depths. Near surface, the two displacement values differ by some 17 percent while near 150 foot depth the difference is some 8 percent. The magnitude of the difference between the upper and lower confidence bounds are somewhat larger than was the case with peak velocity. At 1.67 foot depth, the bounds differ by a factor of about 3.5 while near 150 foot depth, the difference is a factor of about 1.9.

To test the sensitivity of these results to the assumption of a log normal distribution for the upper loading modulus values, two other forms of distribution functions were derived from the WES uniaxial stress-strain data. The first of these was the log uniform distribution which results in all modulus values between an upper and a lower bound value being equally probable (in log space). The second distribution function is the Gamma function which involves non-transformed parameter values and is characterized by a scale factor and a shape factor.

Figure 3-11 compares the three assumed distribution functions in terms of the cumulative fraction of modulus values that are equal to or less than a given value. For example, at 1 Kbar modulus value, the log normal distribution assumption has about 13% of the modulus values equal to or less than this value, whereas the cumulative fractions for the Gamma and Log Uniform distributions are about 19 and 23 percent, respectively. Two points should be noted relative to the new distribution function assumptions: First, that the Gamma and Log Uniform distributions have a somewhat higher fraction of low modulus values than the log normal distribution. Second, that the log uniform distribution assumption restricts the upper modulus values to the range of 0.55 to 7.74 Kbars.

Monte Carlo programs were developed for a programmable calculator that developed the upper loading modulus values by drawing random numbers from the appropriate distribution functions and calculated the peak velocity using the relationships between peak velocity and upper loading modulus that were discussed with Figure 3-11. A nominal sample size of 500 was used in these calculations. Figure 3-12a compares the mean peak velocity values for the

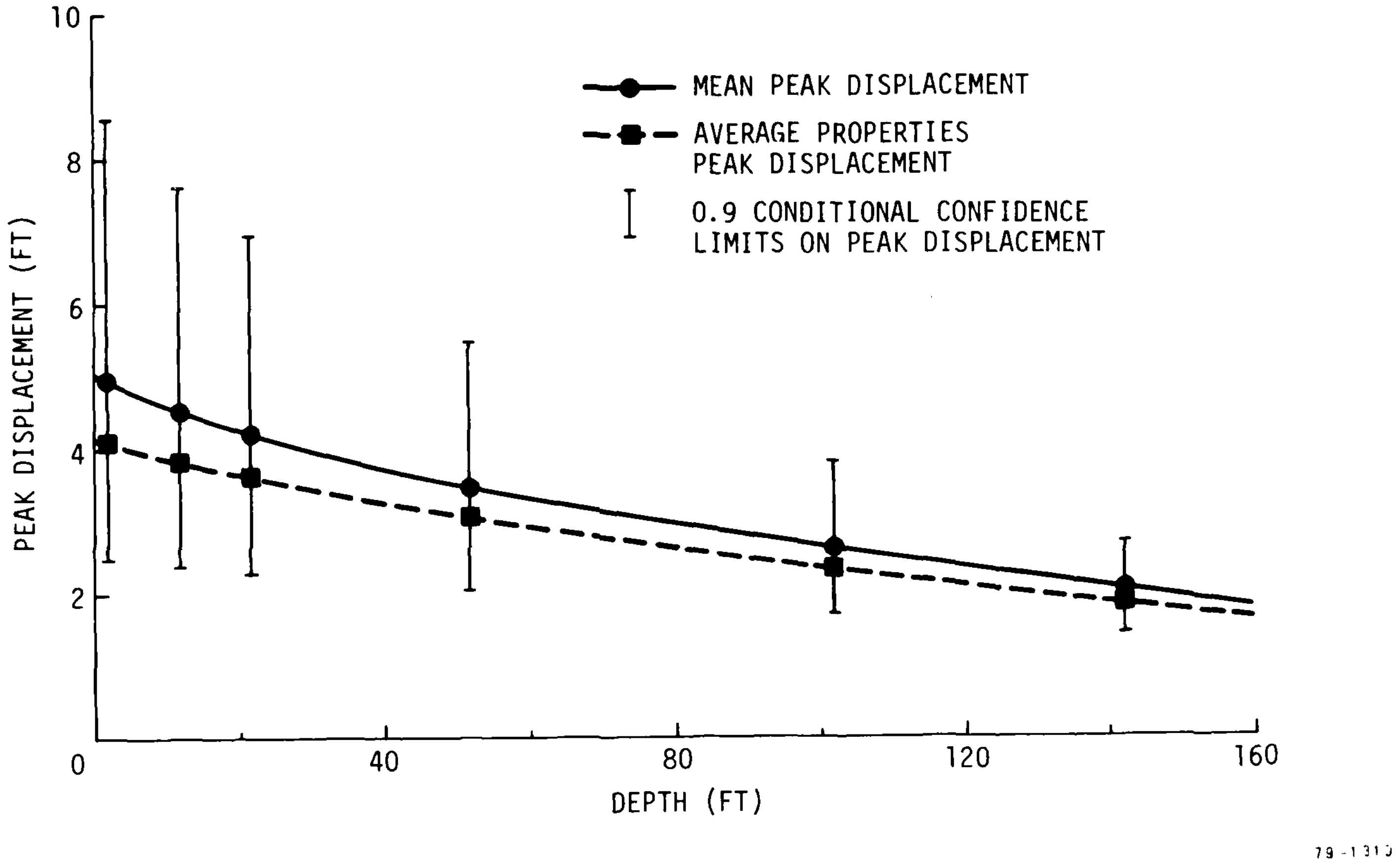


Figure 3-10. Effect of depth on calculated peak displacements.

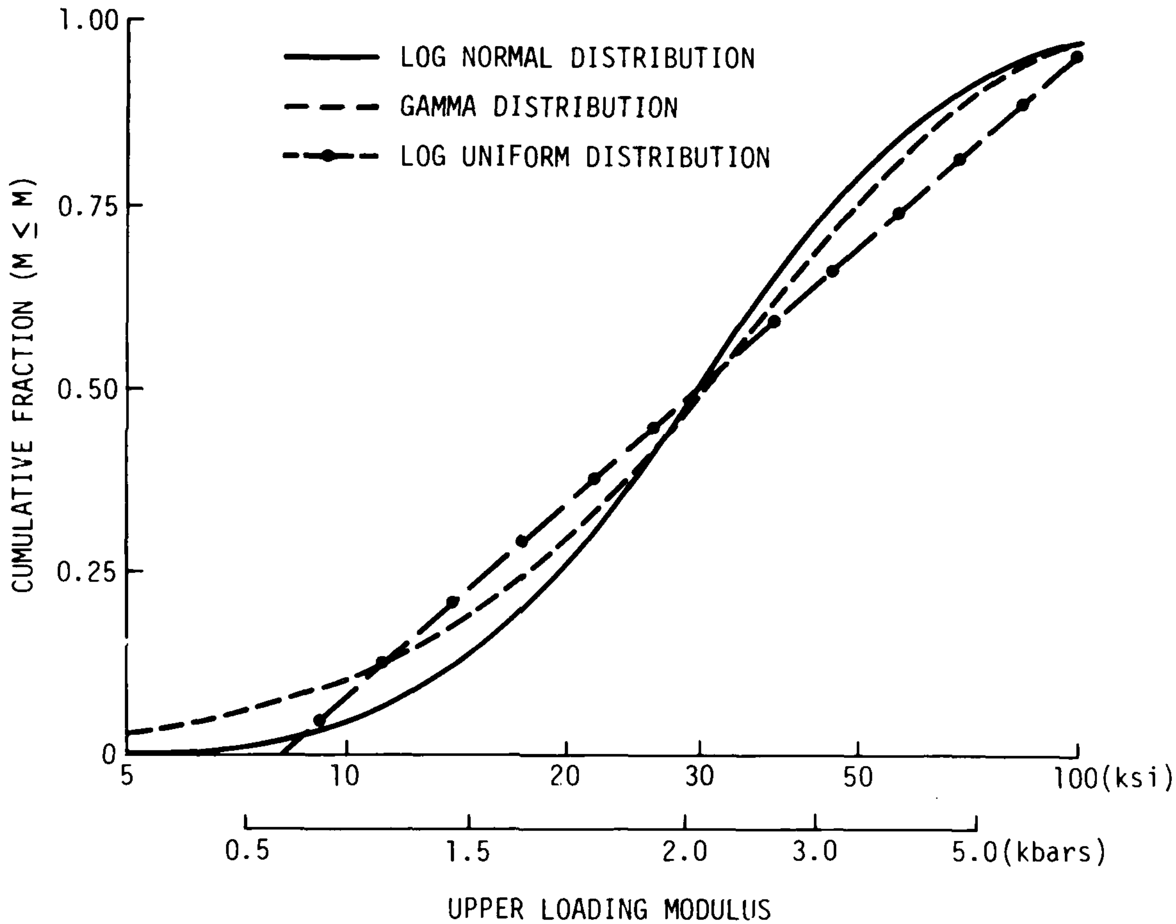


Figure 3-11. Comparison of upper loading modulus distribution assumptions.

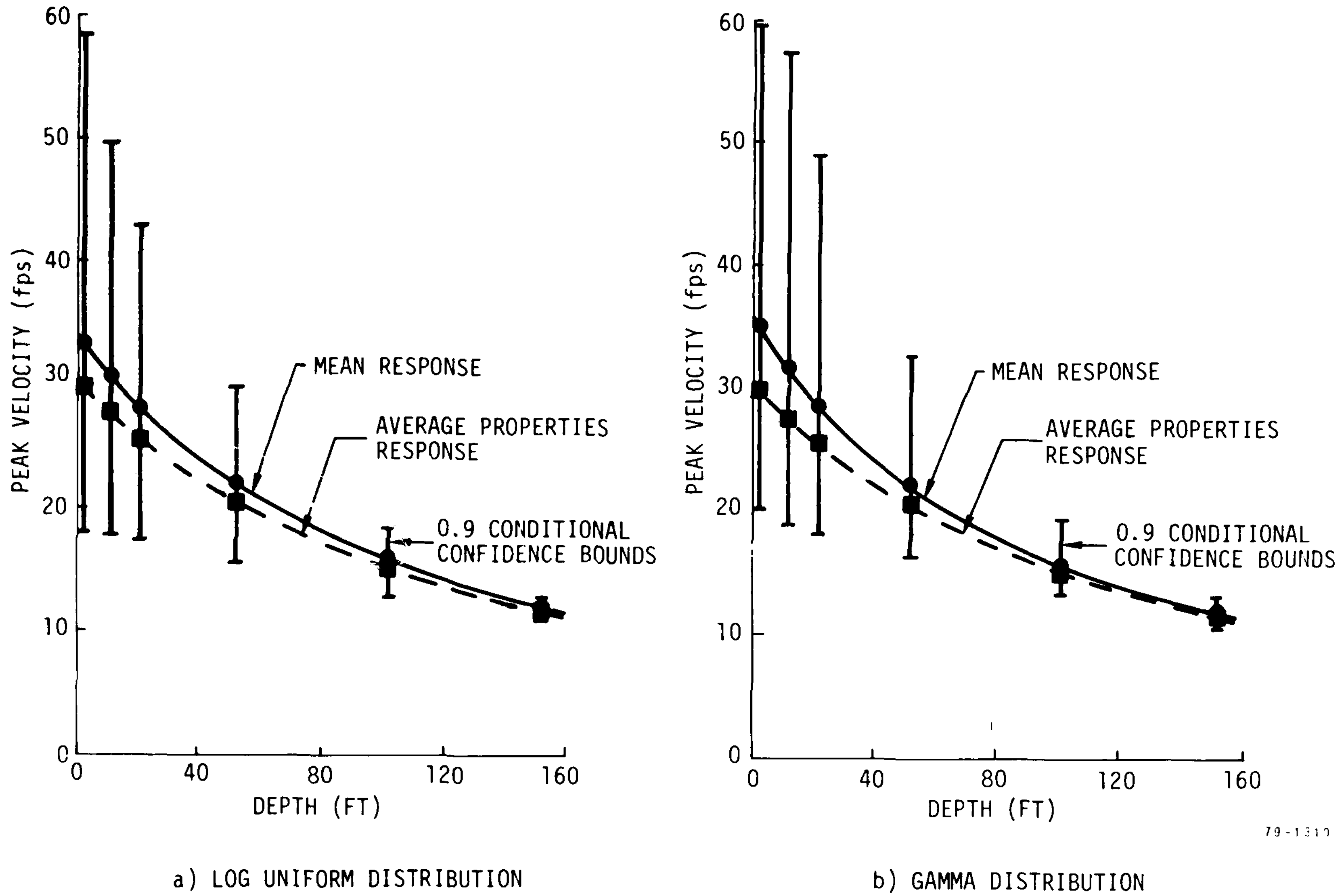


Figure 3-12. Calculated peak velocities for alternate modulus distribution assumptions.

case of the assumed log uniform distribution with the peak velocity values calculated using the average properties. Also shown are the 0.9 conditional confidence bounds produced by the log uniform distribution of the upper loading modulii. As can be seen, the overall results are quite similar to those shown in Figure 3-8 for the case of the log normal distribution assumption. The average properties peak velocity values are always less than the mean velocity but the difference is a maximum of about 10%.

Figure 3-12b shows similar data for the case where the upper loading modulii are assumed to have a Gamma distribution. The results are again quite similar to those obtained with the log normal distribution assumption except the upper confidence bound velocity at any depth is somewhat greater with the Gamma distribution assumption.

Overall, these results suggest that for the case examined, the average properties lead to average results hypothesis appears to be reasonably valid. The average properties response always under-predicts the mean of the distribution of responses, but the percentage difference is relatively small, 17 percent being the maximum difference observed. This is due to the combination of coefficient of variation of the upper loading modulus distribution (~0.70) combining with the exponent of the empirical relationship found between peak response and upper loading modulus (maximum of 0.6) to give response distributions with a maximum coefficient of variation of about 0.4.

3-2 GENERALIZATION OF THE HYPOTHESIS TEST

Whether or not the parameters that produced the results of Section 3-1 are truly representative, or not, is open to question. Figure 3-13, which is redrawn from Reference 4, illustrates the relationship between airslap-induced peak vertical velocity and peak overpressure for a series of HE events at Suffield Experimental Station including Distant Plain 1A, 2A, 3, 5, 6, Prairie Flat and Dial Pack. Also shown on that figure are the regression lines for the relationship between median peak velocity and peak overpressure and the 0.9 confidence prediction bounds. These data support a coefficient of variation of the response distribution at a fixed pressure level that is about 1.4 times that found for the dry sand. The median peak velocity at 1000 psi overpressure is also about 1/3 higher than the value derived from the model used in Section 3-1.

According to Reference 4, the area in which these tests took place is characterized by a fairly complex site profile with a shallow water table whose depth below the surface averages about 24 feet with extremes in the neighborhood of 18 feet and 28 feet. The materials above the water table were complexly layered and very compressible. The representative properties uniaxial stress-strain relations for the near surface materials show loading modulii near a vertical strain level of 1000 psi that range from about 0.4 to 2.75 Kbars. There is sufficient data to further characterize, in a statistical sense, the innate variability of loading modulus values for this site. The data that is available suggests that there may be additional variability of, perhaps, a factor of 1.5 around the represent-

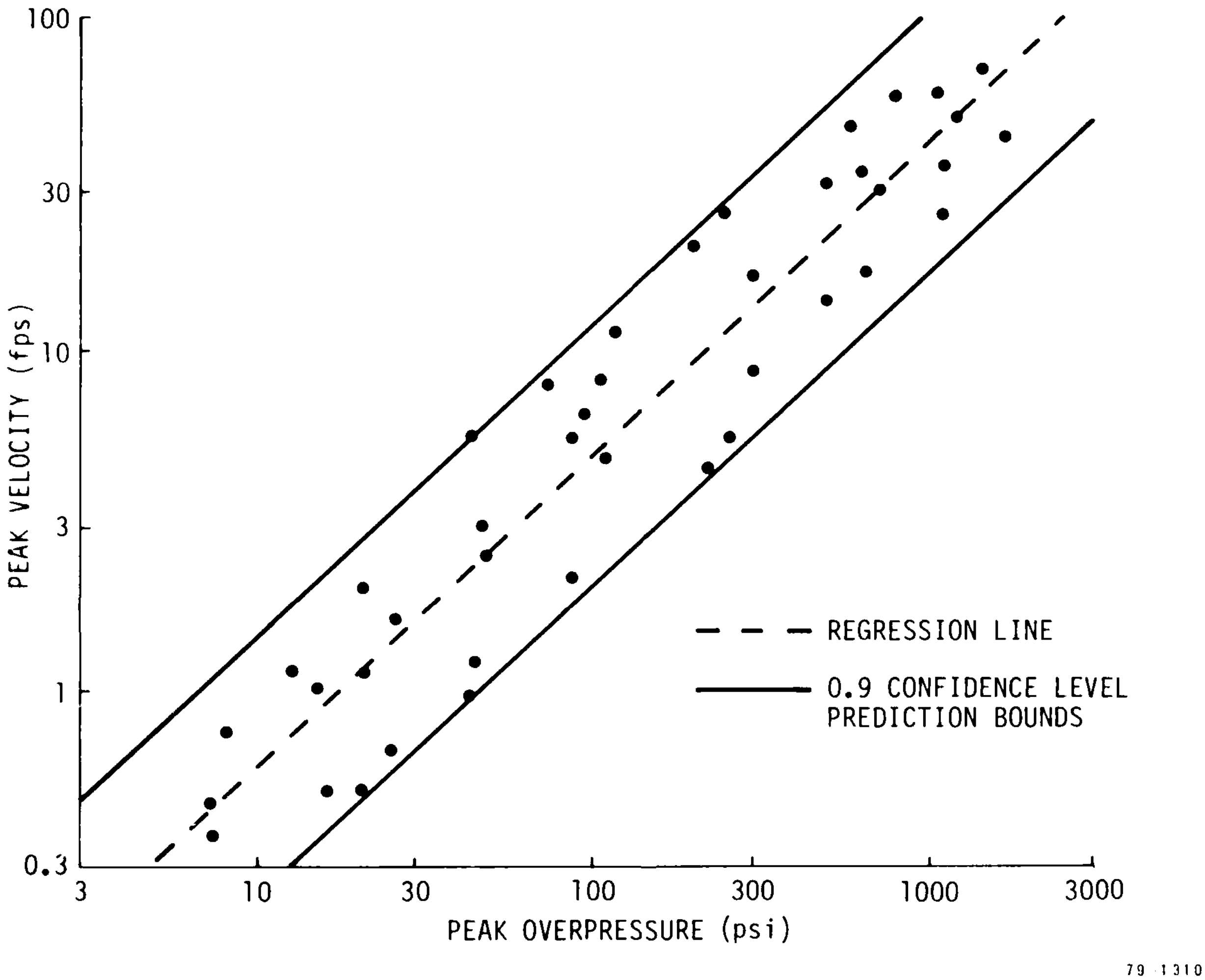


Figure 3-13. Near surface (1.5 ft.) airslap induced peak vertical velocities at Suffield Experimental Station.

ative property values. Overall, the spread in the loading modulii for the near surface materials suggests that the coefficient of variation for these loading modulii is very near unity.

Figure 3-14a illustrates the effect of the coefficient of variation of a material property on the ratio of the average properties response to the mean value of the response for cases where response varied as (property value)$^{\alpha}$ such as was the case for the peak velocity and peak displacement data calculated in this effort where α ranged from nearly zero to about -0.6. The best estimate coefficient of variation of the upper loading modulus for the dry sand was 0.7 which gives the ratio of the average properties' response to the mean value response to a minimum value of about 0.85. If the coefficient of variation of the upper loading modulus were unity, then the minimum value of the ratio of the responses would decrease to about 0.75.

As shown in Figure 3-14b, the behavior of the ratio of the average properties response to the mean response is somewhat different than that for negative exponents. For positive fractional values of the exponent, the average properties' response is always greater than the mean response. Thus, for a parameter such as compression wave velocity, which varies as the square root of the constrained modulus, the average properties result will overestimate the mean value by a few percent.

It is possible to define, for arbitrarily chosen adequacy criteria, regions where the average properties/average results hypothesis produces acceptable results. Figure 3-15 illustrates the regions in value of exponent versus property space where the average properties/average results hypothesis produces a maximum bias in the estimate of the mean response of either 10% (Figure 3-15a) or 30% (Figure 3-15b). For the case of a maximum of 10% bias in the estimate of the mean, exponents between roughly +5 and -4 are admissible when the coefficient of variation of the property value is near 0.1, while for the case of a coefficient of variation near unity, the permissible values of the exponent are restricted to the range of about +5/4 to -1/4. (Note that these bounds are symmetrical around an exponent value of 1/2.) Comparison with the case where 30% bias in the estimate is assumed to be acceptable shows that for a fixed value of the coefficient of variation of the property value, relaxation of the accuracy requirements increases the range of exponents where the hypothesis produces acceptable estimates of the mean response.

Overall, these results suggest that the average properties lead to average results hypothesis may be an adequate estimator for the free field ground shock estimation process for cases where a single parameter dominates the results. The average properties response is a biased estimation of the mean response for all cases examined herein. The degree to which the average properties response is biased depends on the coefficient of variation of the property value and the sensitivity of the response to the property value.

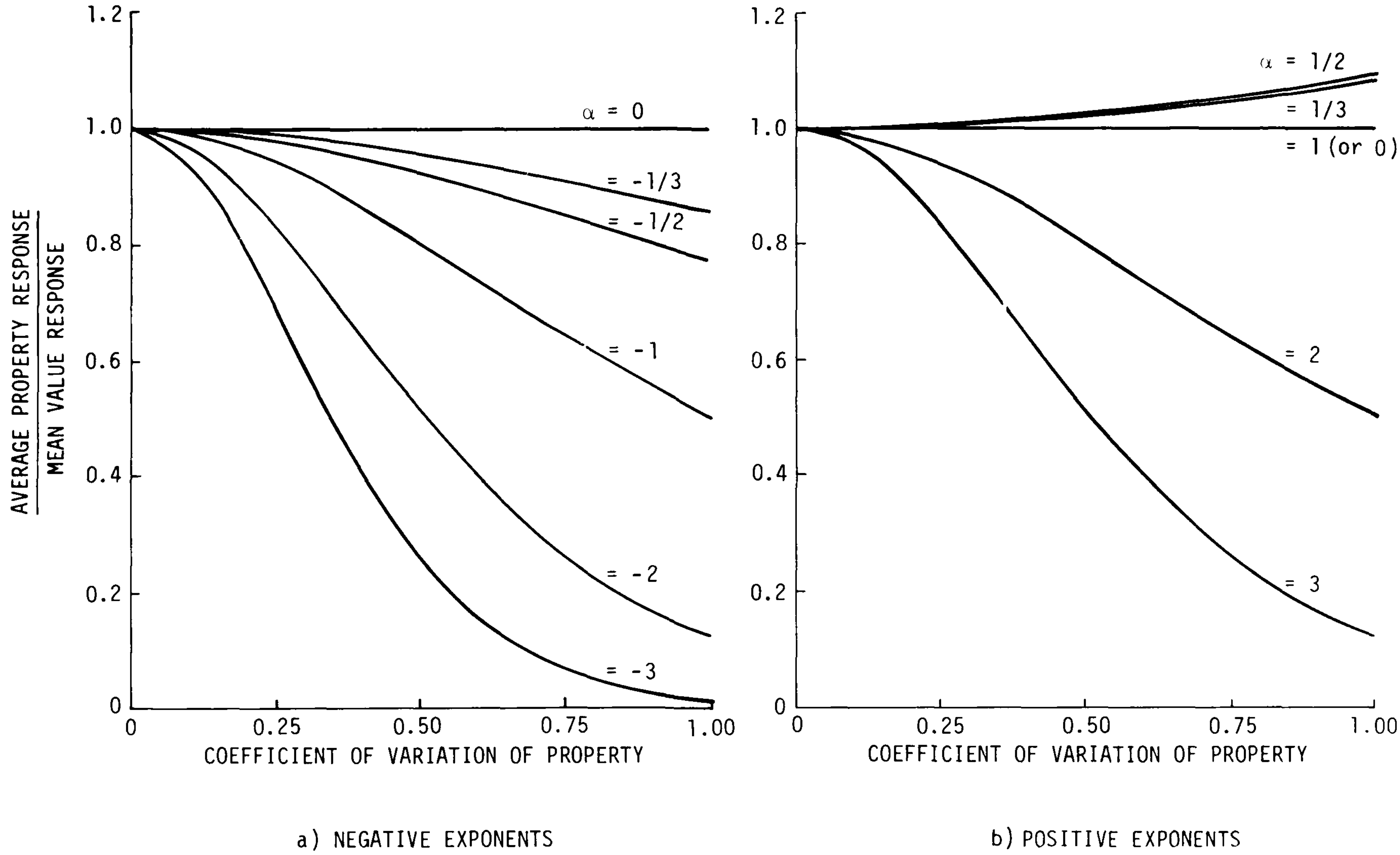


Figure 3-14. Generalization of average properties/average results hypothesis tests.

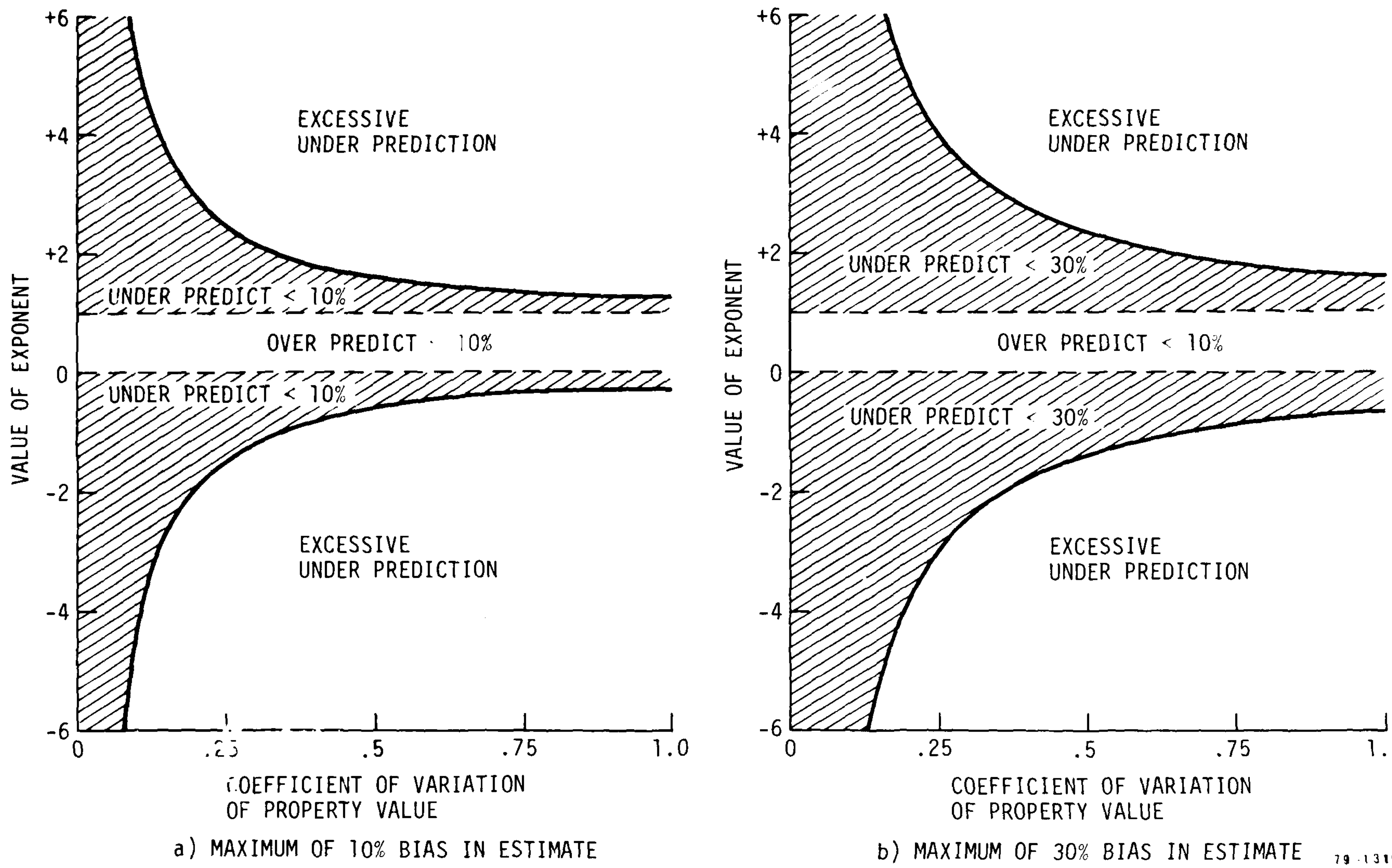


Figure 3-15. Regions of adequacy of average properties/average results hypothesis.

Generalizing these results to cases where the response is controlled by two or more parameters may be dangerous. When there is no correlation between the material properties parameters, the average properties' response will remain a biased estimator of the mean response with a degree of bias that depends on the coefficients of variation of the pertinent materials properties parameters and the degree of sensitivity of the response to the parameter values.

Assuming for example, that a response such as peak velocity depends on two parameters, P_1 and P_2, according to the relationship

$$r = a \cdot P_1^{\alpha_1} \cdot P_2^{\alpha_2} \tag{13}$$

and that P_1 and P_2 are uncorrelated and log normally distributed, then the response will have a log normal distribution with the characteristics

$$\text{Median} = r_{50} = a(P_{10})^{\alpha_1} \cdot (P_{20})^{\alpha_2} \tag{14}$$

$$\text{Std Deviation} = \gamma_{12} = \sqrt{(\alpha_1\beta_1)^2 + (\alpha_2\beta_2)^2} \tag{15}$$

$$\text{Mean} = \bar{r} = r_{50} \exp\left[1/2\ \gamma_{12}^2\right] \tag{16}$$

$$= r_{50} \exp\left[1/2 \left(\alpha_1^2\ \beta_1^2 + \alpha_2^2\ \beta_2^2\right)\right]$$

The average properties response will be given by

$$r_{AP} = a\left[P_{10}e^{\beta_1^2/2}\right]^{\alpha_1} \cdot \left[P_{20}e^{\beta_2^2/2}\right]^{\alpha_2} \tag{17}$$

$$= a\ P_{10}^{\ \alpha_1}\ P_{20}^{\ \alpha_2} \exp\left[1/2 \left(\alpha_1\beta_1^2 + \alpha_2\ \beta_2^2\right)\right]$$

$$= r_{50} \exp\left[1/2 \left(\alpha_1\ \beta_1^2 + \alpha_2\ \beta_2^2\right)\right]$$

Thus, as was the case for a single parameter, the average property's response will be less than the mean response for all negative values of the exponents α_1 and α_2 and for certain combinations of positive and negative values of these exponents.

This implies that it might be necessary to accept a larger bias in the estimate of the mean when the response depends on two parameters. Alternatively, it might be necessary to reduce the regions where the average properties/average results hypothesis produces acceptable results from those illustrated in Figure 3-15.

When correlation exists between the pertinent material properties parameters, the situation is more complicated. Depending on the degrees of correlation, the coefficients of variation, and the sensitivity of the response to the various parameters, the average properties response may become an increasingly more biased estimator of the mean response such as was illustrated in Figure 3-14 for the case of a large coefficient of variation of the property value and high sensitivity of the response to the parameter value.

SECTION 4

SAMPLING AND TESTING STRATEGIES AND REQUIREMENTS

Measurement of the key physical and mechanical properties of earth materials is potentially one of the largest sources of uncertainty in the overall free field ground shock estimation process. A degree of uncertainty in the parameter values should be considered natural, simply due to the innate heterogeneity of the earth materials themselves. This is further compounded by the potential uncertainties and bias errors that may arise in the sampling and testing procedures used to obtain the parameter values.

The degree of uncertainty in the measured value of a physical or mechanical property is, in reality, of interest only to the extent that it contributes to the overall uncertainty of the free field response descriptions. It may be the case that a large degree of uncertainty in a measured value of a parameter may be acceptable if the model of reality being used to estimate the free field environment is relatively insensitive to large changes in the parameter value. On the other hand, it is conceivable that the situation may occur where a very small degree of uncertainty in parameter value will contribute a large degree of uncertainty to the free field response descriptors due to the high degree of sensitivity of the model to changes in the parameter value.

This section of the report is an attempt to illuminate some of the issues involved in devising sampling and testing strategies (or plans) that recognize the innate presence of uncertainties in the properties of earth materials and seek to minimize these uncertainties. The areas to be examined are divided on the basis of the absence or presence of spatial correlation of material property values. The discussion treats the case where only a single parameter value is of interest but can easily be generalized to the case of multiple parameters.

4-1 ESTIMATING MEAN RESPONSE - LINEARITY IN RESPONSE FUNCTIONS

Figure 4-1 illustrates the role of sampling and testing for the case where the response to be estimated is assumed to depend only on the value (or some function of the value) of one physical or mechanical property and the mean response is to be estimated using the average properties lead to average results hypothesis. For this case, the key parameters to be produced by the parameter estimation process are estimates of the mean of the property value and the variance of the estimate of the mean of the property value.

Assuming that spatial independence effects are negligible, the measured property value for any one sample would consist of the following components:

1) The true mean property value which will be denoted as ξ.

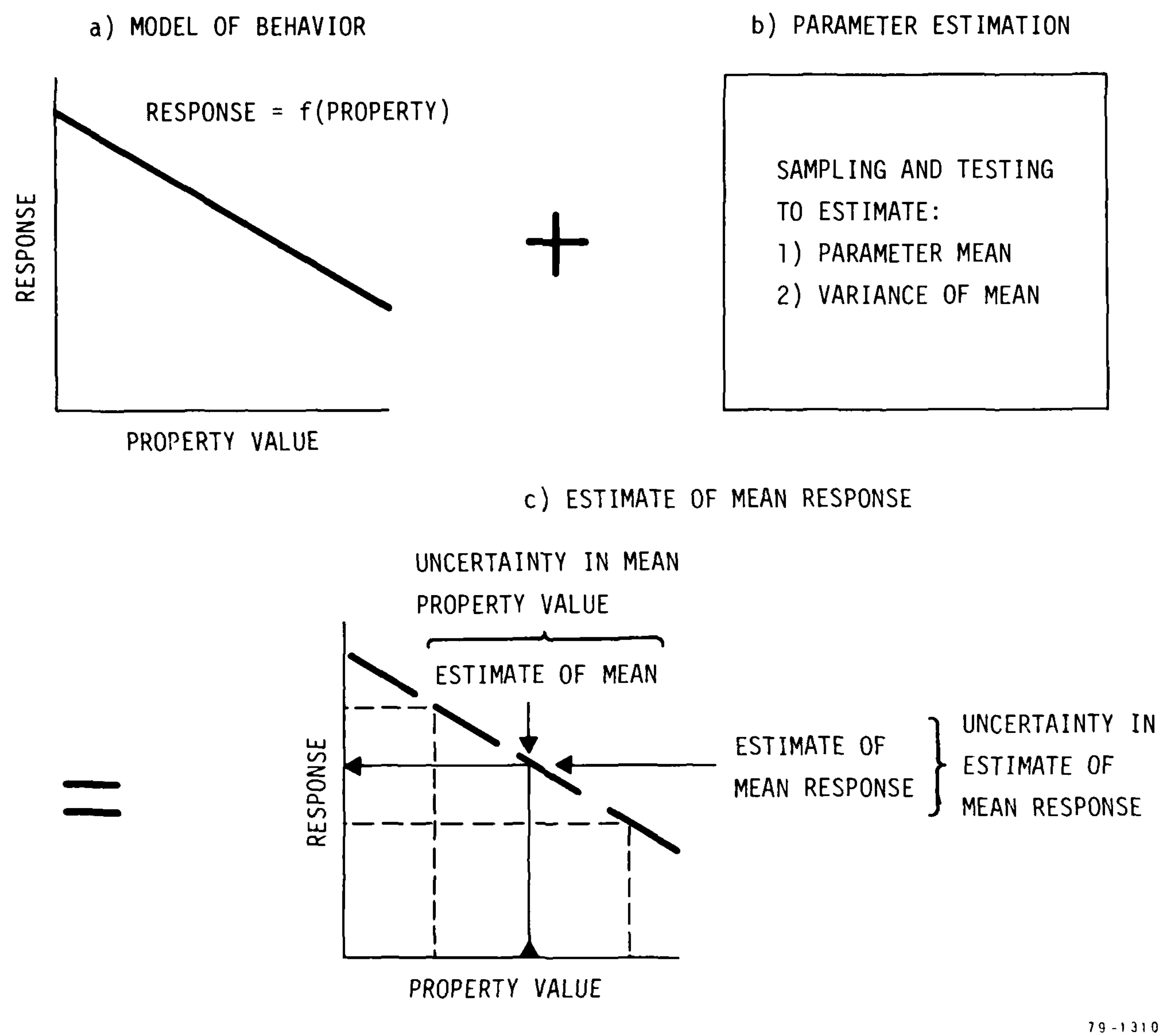


Figure 4-1. Estimating mean response with average properties hypothesis.

2) A random component, $e(i)$, due to the innate variability of the properties which has a mean of zero and a variance of σ_i^2.
3) A random component $e(t)$ due to random errors in sampling and testing which has a mean of zero and a variance of σ_t^2.
4) A systematic, or bias, component $e(b)$ due to sampling disturbances and/or systematic testing errors which will be assumed to have a mean of E_b and a variance of zero.

which can be written as

$$\chi_j = \xi + e(i)_j + e(t)_j + e(b) \tag{18}$$

where the subscript, j, denotes a particular sample. Given N property value measurements, the estimators:

$$\overline{X} = \frac{1}{N}\sum_{i=1}^{N} \chi_i \tag{19}$$

$$s^2 = \frac{1}{N-1}\sum_{i=1}^{N} \chi_i - \overline{X}^{\,2} \tag{20}$$

$$S^2 = \frac{s^2}{N} \tag{21}$$

are not maximum likelihood unbiased estimators of the true mean value of the property (ξ), variance (σ_i^2), and the variance of the estimated mean because of the bias error and random testing error components. An unbiased estimate of the mean and the variance of estimated mean would be obtained by substituting the quantity $\chi_i - E_b$ for the χ_i in the estimation equations but this is, in practice, difficult to impossible since the existence of bias errors is generally only suspected and not quantifiable to the extent of truly estimating the mean bias error in a statistical sense. Therefore, we must assume, with the caveat of "beware of bias errors", that:

a) $\overline{X} \left(= \frac{1}{N} \sum X_i\right)$ estimates the mean property value ξ

b) $s^2 \left(= \frac{1}{N} \sum \left(X_i - \overline{X}\right)^2\right)$ estimates the sum of the variance of the property

population (σ_i^2) and the variance of the random sampling and testing error component (σ_t^2)

c) $S^2 \left(= \frac{s^2}{N}\right)$ estimates the variance of the estimate of the mean,

and further note that there is, in general, no statistic that provides a maximum likelihood estimate of the coefficient of variation of the parameter.

Confidence limits on the estimate of the mean are established using the relationship

$$P_r \left[t_{P_1} \cdot \frac{s}{\sqrt{N}} < \bar{X} - \xi < t_{P_2} \frac{s}{\sqrt{N}} \right] = P_2 - P_1 \tag{22}$$

where P_1 and P_2 define the confidence interval and t is the appropriate value of the "Students - t" distribution with N-1 degrees of freedom which has the property of approaching the normal distribution as the number of degrees of freedom become infinite. This relationship implies that even with random sampling and testing errors, the mean value (ξ) can be estimated to any degree of precision desired by increasing the number of samples that are tested. In order to get an a priori estimate of the sample size required to obtain a given precision of estimate, a distribution function and a value of the variance must be assumed. An expected number of samples required for a given precision of estimate of the mean can then be calculated which will be a rough measure of the actual number of samples required for a given precision of estimate of the mean.

Table 4-1 shows the expected number of samples required to have 0.9 confidence that the estimated mean is within fixed percentages of the actual mean vs the coefficient of variation of the parameter for the case where the random sampling errors are negligible. As should be expected, precise estimates of the mean of parameters with large coefficients of variation require large sample sizes.

Table 4-1. Sample size requirements for estimate of parameter mean. Samples drawn from a normal distribution. No random testing error component.

Coefficient Of Variation Of Parameter	Expected Number Of Samples Required — Precision Of Estimate Of Mean ±10%	±30%	±50%
0.2	10.8	1.2	0.4
0.4	43.3	4.8	1.7
0.6	97.4	10.8	3.9
0.8	173.2	19.2	6.9
1.0	270.6	30.1	10.8

The shear numbers problem may be mitigated in certain cases by the results shown in Figure 3-14 of Section 3-2. The average properties/average results hypothesis is adequate at large coefficients of variation over a limited range of exponents centered around positive 1/2. Since precision of the estimate of the mean response is roughly equal to the exponent times the precision of the estimates of the mean property values, ±30% precision of the estimate of the mean property value would produce uncertainties in the estimated mean response that are comparable to the inherent bias of the average property/average results hypothesis if the exponent were between, say, plus 1/2 and minus 1/2. Thus, sample sizes in the order of a small multiple of ten are probably adequate for exponents in this regime.

Table 4-2 illustrates the effect of random testing errors on this conclusion. As can be seen, for a fixed coefficient of variation of the property, the sample size requirements increase linearly with the ratio of the testing variance to the sample variance. Thus, twice as many samples are going to be required for a fixed precision if the random testing errors are of the same order as the innate variability of the parameter being tested.

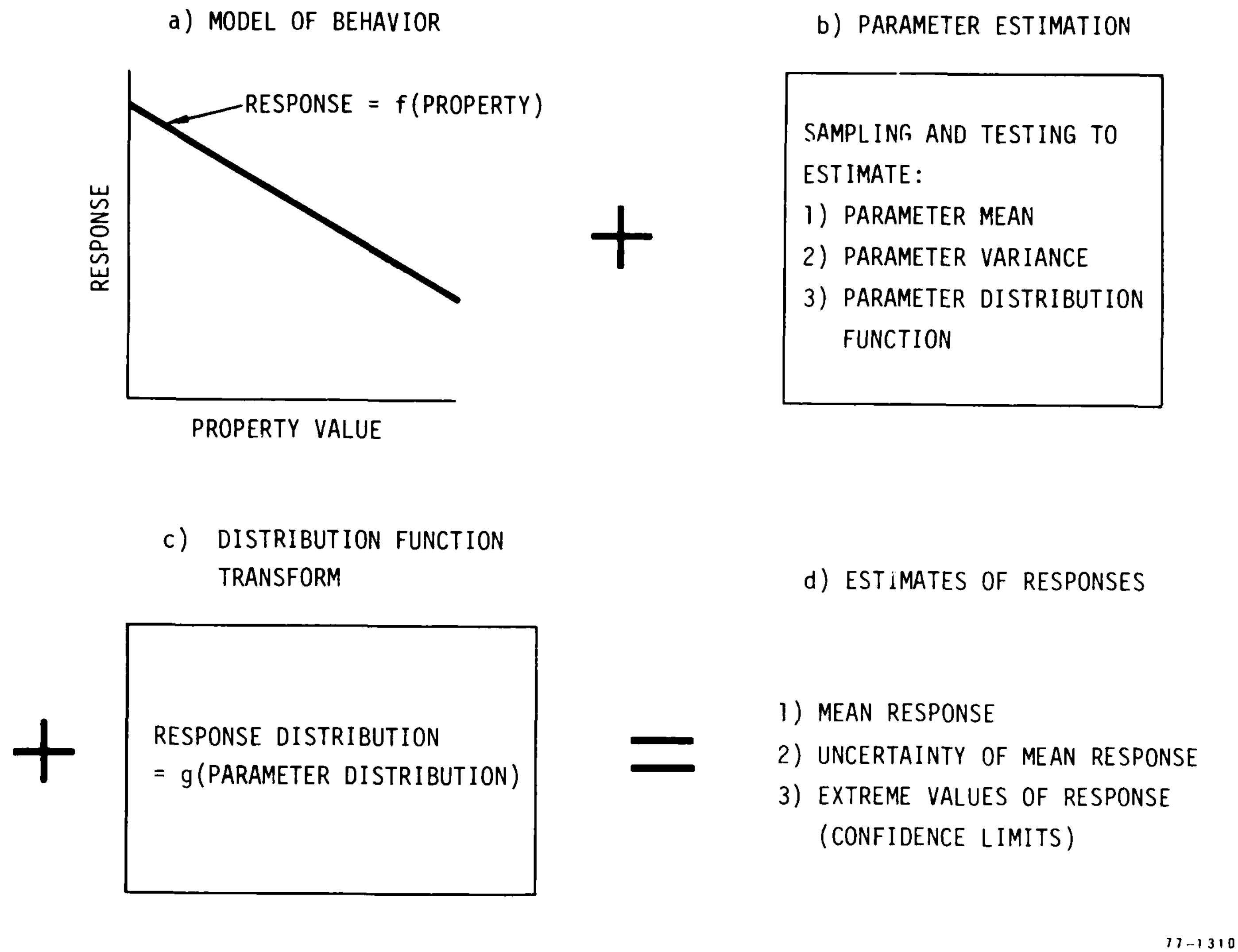


Figure 4-2. Estimating mean response without average properties hypothesis.

Table 4-2. Effect of random testing errors on sample size requirements. Samples drawn from a normal distribution. ± 30% precision of estimate of mean.

Coefficient Of Variation Of Parameter	Expected Number of Samples Required		
	$\frac{\text{Testing Variance}}{\text{Parameter Variance}}$ =		
	0	0.5	1.0
0.2	1.2	1.8	2.4
0.4	4.8	7.2	9.6
0.6	10.8	16.8	21.7
0.8	19.2	28.9	38.6
1.0	30.1	45.1	60.3

Overall, these results indicate that sample sizes of the order of a small multiple of ten may provide an adequate estimate of the mean value of a parameter when the average properties lead to average hypothesis results is used to estimate the mean response. Sampling and testing errors may, perhaps, add a factor of 2 to the sample size requirements. The uncertainty in the predicted mean response will then be of the same order as the inherent bias in the estimated mean response.

4-2 ESTIMATING MEAN RESPONSE - AVERAGE PROPERTIES/AVERAGE RESULTS HYPOTHESIS INADEQUATE

Situations where the average properties/average results hypothesis would produce sufficiently biased estimates of the mean response as to be unusable are not difficult to visualize but it is not known at this time whether any of these occur in the overall free field ground shock estimation process. It is useful, however, to consider the implications of these on sampling and testing strategies should they be found to occur.

Consider, for example, a case similar to those discussed in Section 3, where the model of behavior was response varying as property value to an exponent and the variability of the property value was described by a log normal distribution with median P_{50} and variance β^2. These assumptions lead to the response values being log normally distributed with a median of R_{50} and the variance γ^2 where

$$R_{50} = \text{constant} \cdot P_{50}^{\alpha} \tag{23}$$

and

$$\gamma^2 = \alpha^2\beta^2 \tag{24}$$

Because of the properties of log normal distributions, the mean response is given by

$$\bar{R} = R_{50} \exp [1/2\gamma^2] \tag{25}$$

which involves both the median and the variance of the assumed log normal distribution of the response values.

Establishing confidence limits on this estimate is much more difficult than for the case where the average properties/average results hypothesis was adequate. An approximate estimate can be made, at no specificable confidence level, with the relationship

$$\frac{\Delta\bar{R}}{\bar{R}} = \left[\left(\frac{\Delta R_{50}}{R_{50}}\right)^2 + \left(1/2\gamma^2 \quad \frac{\Delta(\gamma^2)}{\gamma^2}\right)^2\right]^{\pm 1/2} \tag{26}$$

where ΔR_{50} is the uncertainty in the median of the distribution and $\Delta(\gamma^2)$ is the uncertainty in the variance of the distribution, which must be estimated through their functional relationships with the property value distribution descriptors, and the plus sign defines the upper bound limit while minus sign defines the lower bound limit.

The median of the response distribution and the uncertainty in this estimate is obtained by first applying equations (19) through (22) to the algorithms transform of property value data, then transforming into property value space through the relationship

$$P_{50} = \exp [\overline{\ln P}] \tag{27}$$

and then applying equation (23). The variance of the transformed property data has already been estimated above by equation (20) but this estimate includes a random testing error component. If the random testing error component is of negligible magnitude, confidence limits on the estimate of the variance can be made with the relationship

$$P_r \left[\frac{s^2(N-1)}{\chi^2_{P_2}} < \beta^2 < \frac{s^2(N-1)}{\chi^2_{P_1}}\right] = P_2 - P_1 \tag{28}$$

where s^2 is the estimate of the variance, β^2 is the true variance, P_1 and P_2 define the confidence interval, and χ^2 is the so-called "Chi-Squared" statistic with N-1 degrees of freedom. The transformed response variance (γ^2) can now be obtained from the transformed property value variance (β^2) through equation (25) and the mean response then estimated from equation (26).

Inherent in this procedure is an implication of requirement for large sample sizes in the sampling and testing process. Inadequacy of the average properties/average results hypothesis implies fairly large absolute values of the exponent α and/or large coefficients of variation for the property values. From Table 4-1 we saw that precise estimates of the mean of a distribution with a large coefficient of variaton implied sample sizes in the order of hundreds. Table 4-3 illustrates the effect of sample size on the precision of the estimate of the variance. Here a sample size of more than 100 is required for a precision of estimate of the variance of ±20 percent.

Table 4-3. Effect of sample size on the precision of estimate of sample variance.

Sample Size	0.9 Confidence Limits on Ratio of Estimate of Variance To Actual Variance
5	0.18 - 2.37
10	0.37 - 1.88
20	0.53 - 1.58
50	0.64 - 1.35
100	0.79 - 1.24

The situation relative to sample size may not be quite this bleak, however. Assume a simple case where response is inversely proportional to the property value (i.e., $\alpha = -1$) and the property has a coefficient of variation of 0.8. A sample size of twenty, produces approximate 0.9 confidence bounds on the estimate of the mean response of ±40 percent. Increasing the sample size to 50 reduces the confidence bounds to about ±25 percent, while a sample size of 100 reduces the approximate confidence bounds to about ±15 percent. Thus, sample sizes of 100 or less may be adequate for this case.

Overall, estimating mean response without the average properties/average response hypothesis creates more stringent requirements on sampling and testing strategies. Sample sizes on the order of one hundred are probably required to adequately estimate the mean response. Contrawise to the case where the hypothesis was adequate, the magnitude of any random sampling and testing error is important and must be minimized in magnitude if sensible results are to be obtained.

4-3 SAMPLING AND TESTING BUDGET WITH SPATIAL CORRELATION

The presence of spatial correlation of materials properties of earth materials discussed in Reference 3 suggests that if a group of small subareas, with linear dimensions that are small compared to the spatial correlation distances, were defined, at distances that are large compared to the spatial correlation distance, and the mean and variance of

some physical property such as a near surface loading modulus were measured for each of the subareas that each of these values would be different. Reference 6 defines a model of statistical behavior that perhaps adequately models this type of behavior.

It is assumed that the property value of the jth sample taken from the ith subarea consists of the components

$$X_{ij} = \xi + Y_i + Z_{ij} \tag{29}$$

where ξ is the true mean property value for the total area being tested, Y_i is a random variable from a normal distribution with mean zero and variance ω^2, and Z_{ij} is a random variable drawn from a normal distribution with a mean zero and variance of σ^2. The quantity ω^2 will be referred to as the variance of the mean while σ^2 will be referred to as the local variance. The local variance includes any random sampling and testing error components. Given n samples from each of k subareas the statistic

$$\bar{X}_i = \frac{1}{N}\sum_{j=1}^{N} X_{ij} \tag{30}$$

estimates the mean of the property value for the ith subarea while

$$\bar{X} = \frac{1}{K}\sum_{i=1}^{h} \bar{X}_i \tag{31}$$

estimates the mean of the property value over the total area being considered. The statistic

$$s_1^2 = \frac{\sum_{i=1}^{k} X_{ij} - \bar{X}_i^{\,2}}{k(n-1)} \tag{32}$$

provides an estimate of the variance component σ^2, while

$$s_2^2 = \frac{\sum_{i=1}^{k} (\bar{X}_i - \bar{X})}{k-1} \tag{33}$$

is an estimate of the quantity $\sigma^2 + n\omega^2$ and

$$V[\bar{X}] = \frac{S_2^2}{nk} \tag{34}$$

is the variance of the estimate of the mean for the total area.

Suppose that instead of subareas, that we are dealing with bore holes and that the cost to drill a bore hole is C_1. Also assume the cost to prepare and test each sample from a bore hole is C_2. The total cost of the measurement program involving k bore holes and n samples from each bore hole will be

$$C = kc_1 + knc_2 \tag{35}$$

The expected value of the variance of the estimated mean is given by

$$V[\bar{X}] = \frac{\sigma^2 + n\omega^2}{kn} \tag{36}$$

so that solving equation (35) for k and substituting into (36)

$$V[\bar{X}] = \left(\frac{\sigma^2 + n\omega^2}{n}\right)\left(\frac{c_1 + nc_2}{c}\right) \tag{37}$$

Manipulation of this function reveals that the minimun variance estimate of the mean occurs when

$$n^* = \sqrt{\frac{\sigma^2}{\omega^2} \cdot \frac{c_1}{c_2}} \tag{38}$$

with n* restricted to values equal to or greater than unity. Figure 4-3 illustrates the behavior of this optimum number of samples per hole with changes in the ratio of the costs and changes in the ratio of variance components. As should be suspected from the form of equation (38), when measurements cost a significant fraction of the cost of a borehole, the optimum number of measurements per borehole is small, irrespective of the ratio of the variances. Conversely, as the cost per measurement approaches a small fraction of the cost per borehole, the optimum number of measurements depends more strongly on the ratio of variances.

In parallel with an optimal number of measurements per borehole, there is also an optimal allocation of cost resources between drilling costs and measurement costs. Factoring equation (35) and substituting for n from (38), we have

$$C = kc_1 \left(1 + n^* \frac{c_2}{c_1}\right) \tag{39}$$

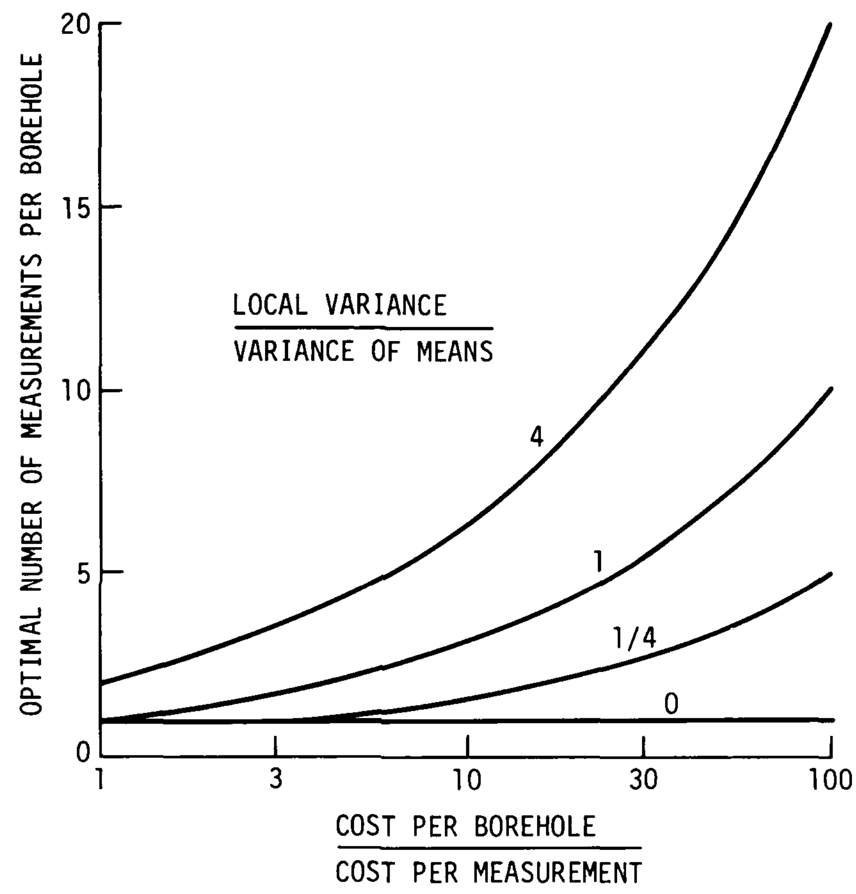


Figure 4-3. Definition of optimal number of measurements per borehole.

as the total exploration cost. The term within the bracket does not involve k, the number of boreholes. Therefore, irrespective of the total cost, the fraction of the cost resources that should be devoted to measurements is

$$f_M = \frac{n^* \frac{c_2}{c_1}}{1 + n^* \frac{c_2}{c_1}} \qquad (40)$$

Figure 4-4 shows the effect of the cost ratio and the variance ratios on this allocation fraction. As might be expected when the cost per measurement of the same magnitude as the cost per borehole (i.e., $\frac{c_1}{c_2} \sim 1$) as is probably the case for the in situ CIST test, the optimal allocation involves about half the resources being allocated to measurements. As the cost per measurement decreases relative to the cost per borehole, the fraction of the resources allocated to measurements decreases and becomes, in a percent-wise basis, increasingly more sensitive to the variance ratio reflecting the sensitivity of the optimum number of measurements per borehole to this ratio. At the other extreme, at cost ratios in the realm of 30 to 100, as might be the case for laboratory testing, the optimal allocation of resources involves a maximum of 30% of the resources being allocated to measurements.

Turning next to the question of sample size requirements, the precision of the estimate of the mean is calculated with equation (22) in exactly the same manner as was the case with no spatial correlation effects. The expected number of boreholes required to yield a given precision of estimate of the mean, however, depends on factors in addition to the coefficient of variaton of the property being measured.

Table 4-4 illustrates the effect of the precision of the estimate of the mean (at the 0.9 confidence level) and the coefficient of variaton of the parameter being estimated on the expected number of boreholes required for the case where the local variance is equal to the variance of the mean and the cost ratio is either 4 or 100. As was the case when there was no spatial correlation of parameter values, precise estimates of the mean of a parameter with a large coefficient of variation requires a large sample size.

The total number of measurements to be made is, in this case, however, not solely determined by the precision required and the coefficient of variation of the parameter. For example, with a coefficient of variation of 0.6 and a ±10 percent precision level, the number of boreholes required at a cost ratio of 4 is about 73 compared to the 54 required when the cost ratio is 100. The optimal number of samples per borehole for the two cost ratios from Figure 4-4 are 2 and 10, respectively, leading to 146 parameter measurements

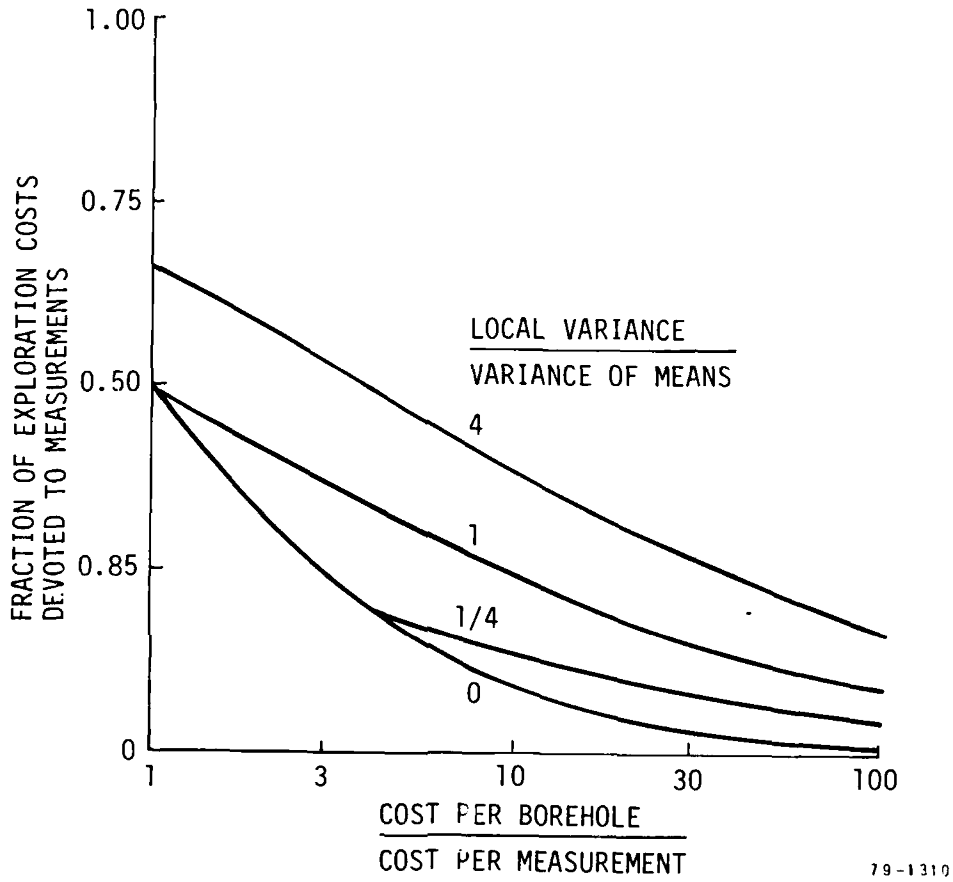


Figure 4-4. Optimal allocation of exploration costs.

when the cost ratio is 4 and 540 measurements when the cost ratio is 100. For comparative purposes, when there is no spatial correlation of properties, Table 4-4 shows a requirement of just under 100 measurements for the same level of precision in the estimate of the mean.

The effect of the variance ratio on the expected number of boreholes required for a ±10 percent precision of estimate of the mean is illustrated in Table 4-5 using the same cost ratios as discussed above. When the variance ratio is zero, no spatial correlation exists and the results are identical to those shown in Table 4-1. With non-zero variance ratios (i.e., with spatial correlation of the parameter being measured), the expected number of boreholes required is sensitive to both the variance ratio and the cost ratio. This behavior should be expected since increasing either the variance ratio or the cost ratio leads to a larger number of measurements per borehole under the optimization that minimizes the variance of the estimate of the mean.

The actual number of measurements to be made, however, does not change radically when the variance ratio changes from 1 to 4. Again, considering the case where the parameter has a coefficient of variation of 0.6, the number of measurements made with a variance ratio of 4 is about 160 for the lower cost ratio and about 470 for the higher cost ratio compared to the previously discussed values of about 150 and 540 when the variance ratio is unity. These results can be extended to other precision of estimate of the mean values through a square relationship increasing the precision of the estimate to, say 5 percent (i.e., a factor of 2), increases the number of boreholes and total measurements required by a factor of four while decreasing the precision of the estimate to, say, 30% decreases the number of boreholes, and total measurements, required by a factor of nine.

Considering the case where the mean response is to be estimated using the average properties/average results hypothesis, these results suggest that a sampling and testing program involving a small multiple of ten boreholes may be adequate if the coefficient of variation of the property value is relatively small or, by the argument of Section 4-1, if the exponent of the response parameter value relationship is small. The number of measurements made will, of course, depend on the variance and cost ratios in the manner previously discussed, but should be somewhere in the range of a small multiple of 25.

Turning next to the case where the average properties/average results hypothesis is inadequate to estimate the mean response, the situation is somewhat more bleak than was discussed in Section 4-2 for the case of no spatial correlation of materials properties. The procedure used to estimate the mean response and the uncertainty in the mean response, however, is identical to that discussed in Section 4-2.

The estimators of the various parameters required to determine the uncertainty in the mean response are different in this case. The mean property value is estimated by equation (31) and the variance of this estimate is determined from equation (34). These two

Table 4-4. Expected number of boreholes required.

Local Variance = Variance of Mean

$$\frac{\text{Borehole Cost}}{\text{Measurement Cost}} = 4 \text{ (or 100)}$$

Coefficient Of Variation Of Parameter	Expected Number of Boreholes Required		
	Precision of Estimate of Mean		
	±10%	±30%	±50%
0.2	8.1 (6.0)*	0.9 (0.7)	0.3 (0.2)
0.4	31.5 (23.8)	3.6 (2.6)	1.3 (1.0)
0.6	73.1 (53.6)	8.1 (6.0)	2.9 (2.1)
0.8	129.9 (95.3)	14.4 (10.6)	5.2 (3.8)
1.0	203.0 (148.8)	22.6 (16.5)	8.1 (6.0)

*Numbers in parenthesis are for cost ratio of 100.

Table 4-5. Expected number of boreholes required.

±10 Percent Precision of Estimate of Mean

$$\frac{\text{Borehole Cost}}{\text{Measurement Cost}} = 4 \text{ (or 100)}$$

Coefficient Of Variation Of Parameter	Expected Number of Boreholes Required		
	Variance Ratio		
	0	1	4
0.2	10.8 (10.8)*	8.1 (6.0)	4.3 (2.6)
0.4	43.3 (43.3)	31.5 (23.8)	17.3 (10.4)
0.6	97.4 (97.4)	73.1 (53.6)	39.0 (23.4)
0.8	173.1 (173.1)	129.9 (95.3)	69.3 (41.6)
1.0	270.6 (270.6)	203.0 (148.8)	108.2 (64.9)

*Numbers in parenthesis are for cost of 100.

quantities suffice to determine the quantity $\frac{\Delta R_{50}}{R_{50}}$ of equation (26). The local variance can be estimated using equation (32) and the uncertainty in this estimate is given by equation (22). The variance of the mean is found from the relationship

$$\hat{\omega}^2 = \frac{S_2^2 - S_1^2}{n} \tag{41}$$

where the quantity S_2^2 is found from equation (33). The uncertainty in this estimate of the variance of the mean cannot be estimated in any straight-forward manner. One technique is to assume that the estimate $\hat{\omega}^2$ is normally distributed and use the method that was used to estimate the uncertainty in the local variance. This approximation is poor from values of k less than about 50. Other techniques are discussed in Reference 6, but these also only give approximate uncertainty bounds for the estimate of the variance of the mean. Thus, the variance and the uncertainty in the variance to be used in equation (26) will only be approximations and, hence, the estimate of the uncertainty in the mean response calculated using equation (26) can only be considered a, perhaps, crude approximation.

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